ADAPTIVE FUZZY CONTROL OF DC MOTORS USING STATE AND OUTPUT FEEDBACK

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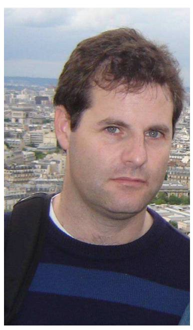
Abstract

Conventional PID of state feedback controllers for DC motors have poor performance when changes of the motor or load dynamics take place. Moreover, neglecting the impact of external disturbances and of nonlinearities may risk the stability of the closed-loop system. To handle these shortcomings adaptive fuzzy control of DC motors is proposed. This paper proposes a method for the control of DC motors, which can be applied to linear or nonlinear models, and which is also robust to uncertainties or external disturbances. Neuro-fuzzy networks are used to approximate the unknown motor dynamics. The information needed to generate the control signal comes from feedback of the full state vector or from feedback of only the system's output. In the latter case a state observer is used to estimate the parameters of the state vector. The stability of the closed-loop system is proved with the use of Lyapunov analysis. The performance of the proposed control approach is evaluated through simulation tests. Comparing to model-based approaches, the advantages of the proposed adaptive fuzzy control are summarized in the following: (i) there is no dependence upon identification of the mathematical model (linear or nonlinear) expressing the dynamics of the DC motor, (ii) since training of the neuro-fuzzy approximators is repeatedly undertaken in every control cycle, any changes to the motor dynamics can be identified online, and hence the control approach is useful for time-varying models, (iii) regarding operation under external disturbances and measurement noise the proposed adaptive fuzzy controller offers improved robustness. Finally, in case that the control is based only on output feedback there is no need to use specific sensors (for instance accelerometers) to measure all elements of the motor's state vector.

Keywords: DC motors, adaptive fuzzy control, state feedback, output feedback, H_∞ tracking, neuro-fuzzy approximators, state observer.

Presenting Author's Biography



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1 Introduction

DC motors are widely used in industrial systems, such as robotic manipulators, because their control is relatively simple and they are reliable for a wide range of operating conditions. DC motors are usually modelled as linear systems and then linear control approaches are implemented. However, most linear controllers have unsatisfactory performance due to changes of the motor/load dynamics and due to nonlinearities introduced by the armature reaction. Neglecting the impact of external disturbances and of nonlinearities may risk the stability of the closed-loop system [1-3]. For the aforementioned reasons DC motor control based on conventional PID or model-based feedback controllers can be inadequate and more effective control approaches are needed.

Recently, there has been considerable effort in the design of nonlinear controllers for high performance servo systems [4-5]. If the nonlinearities of the motor are known functions, then adaptive tracking control methods with the technique of input-output linearization can be used. However, when these nonlinearities or disturbances are unknown, neural or fuzzy control is more suitable for succeeding satisfactory performance of the closed-loop system [6-8]. Many results in the area of neuro-fuzzy control have been obtained [9-12]. The feasibility of applying neuro-fuzzy networks to model unknown dynamic systems has been demonstrated in several studies. Both state feedback and output feedback linearization methods have been presented [13-18]. It has been shown that, output feedback controllers based on state observers can guarantee the global stability of the closed-loop system [19-23].

This paper proposes a method for the control of DC motors, which can be applied to linear or nonlinear models, and which is also robust to uncertainties or external disturbances. The paper extends the results of [16,24]. Two cases can be distinguished: (i) control with feedback of the full state vector, (ii) control using only output feedback. In the first case the closed-loop system consists of the DC motor and an adaptive fuzzy controller based on H_∞ theory [26-28]. Neuro-fuzzy networks are used to approximate the unknown motor dynamics and subsequently this information is used for the generation of the control signal. In the second case the closed-loop system consists of the DC motor, a state observer that estimates the parameters of the state vector from output measurements, and an adaptive fuzzy H_∞ controller that uses the estimated state vector. Neuro-fuzzy estimators are employed as in the first case to approximate the unknown dynamics of the system, but this time they receive as input the estimated state vector [24].

Comparing to model-based approaches, the advantages of the proposed adaptive fuzzy control are summarized in the following: (i) there is no dependence upon identification of the mathematical model (linear or nonlinear) expressing the dynamics of the DC motor, (ii) since training of the neuro-fuzzy approximators is repeatedly undertaken in every control cycle, any changes to the motor dynamics can be identified online, and hence the control approach is useful for time-varying models, (iii) regarding operation under external disturbances and measurement noise the proposed adaptive fuzzy controller offers improved robustness. Finally, in case that the control is based only on output feedback there is no need to use specific sensors (for instance accelerometers) to measure all elements of the motor's state vector.

The structure of the paper is as follows: In Section 2 the model of the DC motor is analyzed. A linear model is derived in case that the motor is controlled by the input (field) voltage, while taking into account the armature reaction a nonlinear model is also introduced. In Section 3 adaptive fuzzy H_∞ control of the DC motor with state feedback is presented. The control concept is based on transforming the tracking problem into a regulation problem. The approximation of the unknown model dynamics with the use of neuro-fuzzy networks, and state feedback is explained. In Section 4 Lyapunov stability analysis is given for the closed-loop system consisting of the DC-motor and the state feedback-based adaptive fuzzy controller. In Section 5, adaptive fuzzy H_∞ control of the DC-motor using only output feedback is presented. The tracking problem is transformed again into a regulation problem and an observer is introduced to estimate the system's state vector. The approximation of the unknown model dynamics with the use of neuro-fuzzy networks, that receive now as input the estimated state vector, is explained. In Section 6, Lyapunov stability analysis is given for the closed-loop system consisting of the DC-motor, the adaptive fuzzy controller, and the state observer. Finally, in Section 8 simulation tests are carried out, to evaluate the performance of both the state feedback and the output feedback controller.

2 The DC motor model

A direct current (DC) motor model converts electrical energy into mechanical energy. The torque developed by the motor shaft is proportional to the magnetic flux in the stator field and to the current in the motor armature (iron cored rotor wound with wire coils). There are two main ways in controlling a DC motor: the first one named *armature control* consists of maintaining the stator magnetic flux constant, and varying (use as control input) the armature current. Its main advantage is a good torque at high speeds and its disadvantage is high energy losses. The second way is called *field control*, and has a constant voltage to set up the armature current, while a variable voltage applied to the stator induces a variable magnetic flux. Its advantages are energy efficiency, inexpensive controllers and its disadvantages are a torque that decreases at high speeds [25]. A linear model that approximates the dynamics of the DC motor is derived as follows: the torque developed by the motor is proportional to the stator's flux and to the armature's current thus one has

$$\Gamma = k_f \Psi K_\alpha I \quad (1)$$

where Γ is the shaft torque, Ψ is the magnetic flux in the stator field, I is the current in the motor armature. Since the flux is maintained constant the torque of Eq. (1) can be written as

$$\Gamma = k_T I, \text{ where } k_T = k_f \Psi K_\alpha \tag{2}$$

Apart from this, when a current carrying conductor passes through a magnetic field, a voltage V_b appears corresponding to what is called electromagnetic force (EMF)

$$V_b = k_e \omega \tag{3}$$

where ω is the rotation speed of the motor shaft. The constants k_T and k_e have the same value. Kirchhoff's law yields the equation of the motor (Fig. 1):

$$V - V_{\text{res}} - V_{\text{coil}} - V_b = 0 \tag{4}$$

where V is the input voltage, $V_{\text{res}} = RI$ is the armature resistor voltage (R denotes the armature resistor), $V_{\text{coil}} = L\dot{I}$ is the armature inductance voltage. The motor's electric equation is then

$$L\dot{I} = -k_e\omega - RI + V \tag{5}$$

From the mechanics of rotation it holds that

$$J\dot{\omega} = \Gamma - \Gamma_{\text{damp}} - \Gamma_d \tag{6}$$

The DC motor model is finally

$$\begin{aligned} L\dot{I} &= -k_e\omega - RI + V \\ J\dot{\omega} &= k_e I - k_d\omega - \Gamma_d \end{aligned} \tag{7}$$

with the following notations

Notation	Significance
L	armature inductance
I	armature current
k_e	motor electrical constant
R	armature resistance
V	input voltage, taken as control input
J	motor inertia
ω	rotor rotation speed
k_d	mechanical dumping constant
Γ_d	disturbance torque

where the armature designates the iron cored rotor wound with wired coils. Assuming $\dot{\Gamma}_d = 0$ and denoting the state vector as $[x_1, x_2, x_3]^T = [\theta, \dot{\theta}, \ddot{\theta}]^T$, a linear model of the DC motor is obtained:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{k_e k_b}{JL} & -\frac{k_e RI}{JL} - \frac{k_d}{J} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \frac{k_e}{JL} \end{pmatrix} V \tag{8}$$

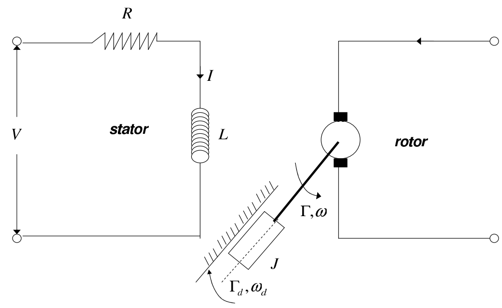

Fig. 1 Parameters of the DC motor model

Usually the DC-motor model is considered to be linear by neglecting the effect of armature reaction or by assuming that the compensating windings remove this effect. Introducing the armature reaction leads to a nonlinear system and in that case a nonlinear model may be appropriate. In that case the dynamic model of the DC-motor model can be written as [7]:

$$x^{(n)} = f(x) + g(x)u \tag{9}$$

with $x^{(n)}$ denoting the i-th derivative of the motor's position (angle) x. The state vector is written as $x = [x_1, x_2, x_3]^T = [\theta, \dot{\theta}, i_\alpha]$, where θ is the position of the motor, $\dot{\theta}$ is the angular velocity of the motor and i_α is the armature current. The functions $f(x)$ and $g(x)$ are vector field functions defined as:

$$f(x) = \begin{pmatrix} x_2 \\ k_1x_2 + k_2x_3 + k_3x_3^2 + k_4T_1 \\ k_5x_2 + k_6x_2x_3 + k_7x_3 \end{pmatrix}, g(x) = \begin{pmatrix} 0 \\ 0 \\ k_8 \end{pmatrix} \tag{10}$$

where $k_1 = -F/J$, $k_2 = A/J$, $k_3 = B/J$, $k_4 = -1/J$, $k_5 = -A/L$, $k_6 = -B/L$, $k_7 = -R/L$, $k_8 = -1/L$, R and L are the armature resistance and induction respectively, and J is the rotor's inertia, while F is the friction.

Now choosing the motor's angle to be the system output, the state space equation of the DC motor can be rewritten as

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= k_1x_2 + k_2x_3 + k_3x_3^2 + k_4T_1\\ \dot{x}_3 &= k_5x_2 + k_6x_2x_3 + k_7x_3 + k_8u \qquad (11)\\ y &= x_1\end{aligned}$$

where T_1 the load torque and u is the terminal voltage. Thus the input-output relation can be written as

$$\begin{aligned}\ddot{x}_2 &= k_1\dot{x}_2 + k_2\dot{x}_3 + 2k_3x_3\dot{x}_3 \ \Rightarrow\\ \ddot{x}_2 &= f(x) + g(x)u\end{aligned}$$

The control approach that will be developed in this paper is a generic one and can be applied to both linear and nonlinear models.

3 Adaptive fuzzy control of the DC motor using state feedback

3.1 Transformation to a regulation problem

The objective is to force the system's output (angle x of the motor) to follow a given bounded reference signal x_d. In the presence of non-gaussian disturbances w, successful tracking of the reference signal is denoted by the H_∞ criterion

$$\int_0^T e^TQedt \leq \rho^2 \int_0^T w^T wdt \qquad (12)$$

where e is the output error and ρ is the attenuation level which corresponds to the maximum singular value of the transfer function $G(s)$ of the linearized equivalent of the system's model [13].

For measurable state vector x and uncertain functions $f(x,t)$ and $g(x,t)$ an appropriate control law for (9) would be

$$u = \frac{1}{\hat{g}(x,t)}[x_d^{(n)} - \hat{f}(x,t) + K^Te + u_c] \qquad (13)$$

with $e^T = [e, \dot{e}, \ddot{e}, \cdots, e^{(n-1)}]^T$, $K^T = [k_n, k_{n-1}, \cdots, k_1]$, such that the polynomial $e^{(n)} + k_1e^{(n-1)} + k_2e^{(n-2)} + \cdots + k_ne$ is Hurwitz. The control law of Eq. (13) results into

$$\begin{aligned}e^{(n)} = -K^Te + u_c + [f(x,t) - \hat{f}(x,t)]+\\ +[g(x,t) - \hat{g}(x,t)]u + \tilde{d},\end{aligned} \qquad (14)$$

where the supervisory control term u_c aims at the compensation of the approximation error

$$w = [f(x,t) - \hat{f}(x,t)] + [g(x,t) - \hat{g}(x,t)]u, \qquad (15)$$

as well as of the additive disturbance $\tilde{d}$. The above relation can be written in a state-equation form. The state vector is taken to be $e^T = [e, \dot{e}, \cdots, e^{(n-1)}]$, which after some operations yields

$$\begin{aligned}\dot{e} = (A - BK^T)e + Bu_c + B\{[f(x,t) - \hat{f}(x,t)]+\\ +g(x,t) - \hat{g}(x,t)]u + \tilde{d}\}\end{aligned} \qquad (16)$$

$$e_1 = C^Te \qquad (17)$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & \cdots & 0\\ 0 & 0 & 1 & \cdots & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \cdots & \cdots & 1\\ 0 & 0 & 0 & \cdots & \cdots & 0\end{pmatrix} \qquad (18)$$

$$B = (0 \quad 0 \quad \cdots \quad \cdots 0 \quad 1)^T$$

$$K = (k_0 \quad k_1 \quad \cdots \quad \cdots \quad k_{n-2} \quad k_{n-1})^T$$

$$C = (1 \quad 0 \quad \cdots \quad \cdots \quad 0 \quad 0)$$

and e_1 denotes the output error $e_1 = x - x_d$. Eq. (16) and (17) describe a regulation problem. The control signal u_c is the H_∞ control term, used for the compensation of $\tilde{d}$ and w

$$u_c = -\tfrac{1}{r}B^TPe \qquad (19)$$

3.2 Approximators for adaptive fuzzy H_∞ control with state feedback

The approximation of functions $f(x,t)$ and $g(x,t)$ of Eq. (9) can be carried out with Takagi-Sugeno neurofuzzy networks of zero or first order (Fig. 2). The estimation of $f(x,t)$ and $g(x,t)$ can be written as

$$\hat{f}(x|\theta_f) = \theta_f^T\phi(x), \ \hat{g}(x|\theta_g) = \theta_g^T\phi(x) \qquad (20)$$

where $\phi(x)$ are kernel functions with elements

$$\phi^l(x) = \frac{\prod_{i=1}^n \mu_{A_i}^l(x_i)}{\sum_{l=1}^L \prod_{i=1}^n \mu_{A_i}^l(x_i)} \ \ l = 1, 2, \cdots, L$$

It is assumed that the weights θ_f and θ_g vary in the bounded areas M_{θ_f} and M_{θ_g} which are defined as

$$\begin{aligned}M_{\theta_f} = \{\theta_f \in R^h : ||\theta_f|| \leq m_{\theta_f}\},\\ M_{\theta_g} = \{\theta_g \in R^h : ||\theta_g|| \leq m_{\theta_g}\}\end{aligned} \qquad (21)$$

with m_{θ_f} and m_{θ_g} positive constants. The values of θ_f and θ_g that give optimal approximation are:

$$\theta_f^* = \text{arg } \min_{\theta_f \in M_{\theta_f}} [sup_{x \in U_x} |f(x) - \hat{f}(x|\theta_f)|]$$
$$\theta_g^* = \text{arg } \min_{\theta_g \in M_{\theta_g}} [sup_{x \in U_x} |g(x) - \hat{g}(x|\theta_g)|]$$

The approximation error of $f(x,t)$ and $g(x,t)$ is given by

$$\begin{gathered} w = [\hat{f}(x|\theta_f^*) - f(x,t)] + [\hat{g}(x|\theta_g^*) - g(x,t)]u \Rightarrow \\ w = \{[\hat{f}(x|\theta_f^*) - f(x|\theta_f)] + [f(x|\theta_f) - f(x,t)]\} + \\ +\{[\hat{g}(\hat{x}|\theta_g^*) - g(x|\theta_g)] + [g(x|\theta_g) - g(x,t)]u\} \end{gathered}$$

where: i) $\hat{f}(x|\theta_f^*)$ is the approximation of f for the best estimation θ_f^* of the weights' vector θ_f, ii) $\hat{g}(x|\theta_g^*)$ is the approximation of g for the best estimation θ_g^* of the weights' vector θ_g. The approximation error w can be decomposed into w_a and w_b, where

$$\begin{gathered} w_a = [\hat{f}(x|\theta_f) - \hat{f}(x|\theta_f^*)] + [\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)]u \\ w_b = [\hat{f}(x|\theta_f^*) - f(x,t)] + [\hat{g}(x|\theta_g^*) - g(x,t)]u \end{gathered}$$

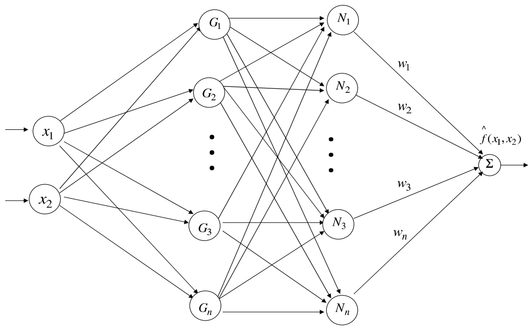

Fig. 2 Parameters of the DC motor model

Finally, the following two parameters are defined: $\tilde{\theta}_f = \theta_f - \theta_f^*$, $\tilde{\theta}_g = \theta_g - \theta_g^*$.

4 Stability of adaptive fuzzy H_∞ control with state feedback

The adaptation law of the weights θ_f and θ_g as well as of the supervisory control term u_c are derived by the requirement for negative definiteness of the Lyapunov function

$$V = \frac{1}{2}e^T Pe + \frac{1}{2\gamma_1}\tilde{\theta}_f^T \tilde{\theta}_f + \frac{1}{2\gamma_2}\tilde{\theta}_g^T \tilde{\theta}_g \tag{22}$$

Substituting Eq. (16) into Eq. (22) and differentiating gives

$$\begin{gathered} \dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{1}{\gamma_1}\tilde{\theta}_f^T \dot{\tilde{\theta}}_f + \frac{1}{\gamma_2}\tilde{\theta}_g^T \dot{\tilde{\theta}}_g \Rightarrow \\ \dot{V} = \frac{1}{2}e^T\{(A - BK^T)^T P + P(A - BK^T)\}e + \\ +B^T Pe(u_c + w + \tilde{d}) + \frac{1}{\gamma_1}\tilde{\theta}_f^T \dot{\tilde{\theta}}_f + \frac{1}{\gamma_2}\tilde{\theta}_g^T \dot{\tilde{\theta}}_g \end{gathered}$$

Assumption 1: For given positive definite matrix Q there exists a positive definite matrix P, which is the solution of the following matrix equation

$$\begin{gathered} (A - BK^T)^T P + P(A - BK^T) - \\ -PB(\tfrac{2}{r} - \tfrac{1}{\rho^2})B^T P + Q = 0 \end{gathered} \tag{23}$$

Substituting Eq. (23) into $\dot{V}$ yields after some operations

$$\begin{gathered} \dot{V} = -\frac{1}{2}e^T Qe + \frac{1}{2}e^T PB(\tfrac{2}{r} - \tfrac{1}{\rho^2})B^T Pe + \\ +B^T Pe(-\tfrac{1}{r}e^T PB) + +B^T Pe(w + d) + \\ \frac{1}{\gamma_1}\tilde{\theta}_f^T \dot{\tilde{\theta}}_f + \frac{1}{\gamma_2}\tilde{\theta}_g^T \dot{\tilde{\theta}}_g. \end{gathered}$$

It holds that $\dot{\tilde{\theta}}_f = \dot{\theta}_f - \dot{\theta}_f^* = \dot{\theta}_f$ and $\dot{\tilde{\theta}}_g = \dot{\theta}_g - \dot{\theta}_g^* = \dot{\theta}_g$. The following weight adaptation laws are considered

$$\dot{\theta}_f = \{ \begin{matrix} -\gamma_1 e^T PB\phi(x) \;\; if \; ||\theta_f|| < m_{\theta_f} \\ 0 \;\; ||\theta_f|| \geq m_{\theta_f} \end{matrix} \tag{24}$$

$$\dot{\theta}_g = \{ \begin{matrix} -\gamma_2 e^T PB\phi(x)u_c \;\; if \; ||\theta_g|| < m_{\theta_g} \\ 0 \;\; ||\theta_g|| \geq m_{\theta_g} \end{matrix} \tag{25}$$

$\dot{\theta}_f$ and $\dot{\theta}_g$ are set to 0, when $||\theta_f|| \geq m_{\theta_f}$, and $||\theta_g|| \geq m_{\theta_g}$ [10]. The update of θ_f stems from a LMS algorithm on the cost function $\frac{1}{2}(f - \hat{f})^2$. The update of θ_g is also of the LMS type, while u_c implicitly tunes the adaptation gain γ_2. Substituting Eq. (24) and (25) in $\dot{V}$ finally gives

$$\begin{gathered} \dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T Pe + e^T PB(w + d) \\ -e^T PB(\theta_f - \theta_f^*)^T \phi(x) - e^T PB(\theta_g - \theta_g^*)^T \phi(x)u_c \\ \Rightarrow \dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T Pe + \\ e^T PB(w + d) + e^T PBw_\alpha \end{gathered}$$

The control scheme is depicted in Fig. 3

Denoting $w_1 = w + d + w_\alpha$ one gets

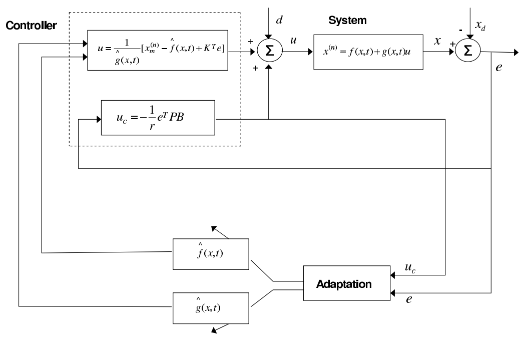

Fig. 3 The proposed H_∞ control scheme in the case of full state feedback

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T Pe + e^T PBw_1$$

or equivalently,

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PBB^T Pe + \frac{1}{2}e^T PBw_1 + \frac{1}{2}w_1^T B^T Pe.$$

Lemma: The following inequality holds

$$\frac{1}{2}e^T PBw_1 + \frac{1}{2}w_1^T B^T Pe - -\frac{1}{2}\rho^2 e^T PBB^T Pe \leq \frac{1}{2}\rho^2 w_1^T w_1 \tag{26}$$

Proof: The binomial $(\rho a - \frac{1}{\rho}b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\begin{gathered}\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \\ \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow \\ ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \\ \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2.\end{gathered}$$

The following substitutions are carried out $a = w_1$ and $b = e^T PB$ and the previous relation becomes

$$\frac{1}{2}w_1^T B^T Pe + \frac{1}{2}e^T PBw_1 - -\frac{1}{2\rho^2}e^T PBB^T P_2 e \leq \frac{1}{2}\rho^2 w_1^T w_1$$

The previous inequality is used in $\dot{V}$, and the right part of the associated inequality is enforced

$$\dot{V} \leq \frac{1}{2}e^T Qe + \frac{1}{2}\rho^2 w_1^T w_1 \tag{27}$$

Hence, the H_∞ performance criterion is satisfied. The integration of $\dot{V}$ from 0 to T gives

$$\begin{gathered}\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||^2 dt + \frac{1}{2}\rho^2 \int_0^T ||w_1||^2 dt \Rightarrow \\ 2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||w_1||^2 dt.\end{gathered}$$

It is assumed that there exists a positive constant $M_w > 0$ such that $\int_0^\infty ||w_1||^2 dt \leq M_w$. Therefore one gets

$$\int_0^\infty ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_w \tag{28}$$

Thus, the integral $\int_0^\infty ||e||_Q^2 dt$ is bounded and according to Barbalat's Lemma $lim_{t\to\infty} e(t) = 0$.

5 Adaptive fuzzy control of the DC motor with output feedback

5.1 Transformation to a regulation problem

For measurable state vector x of the DC-motor and uncertain functions $f(x,t)$ and $g(x,t)$ an appropriate control law for (9) is given by Eq. (13). When an observer is used to reconstruct the state vector x of Eq. (13), the control law of Eq. (13) is written as

$$u = \frac{1}{\hat{g}(\hat{x},t)}[x_m^{(n)} - \hat{f}(\hat{x},t) + K^T e + u_c] \tag{29}$$

The following definitions are used: i) error of the state vector $e = x - x_m$, ii) error of the estimated state vector $\hat{e} = \hat{x} - x_m$, iii) observation error $\tilde{e} = e - \hat{e} = (x - x_m) - (\hat{x} - x_m)$. Applying Eq. (29) to Eq. (9), after some algebraic operations, results into

$$\begin{gathered}x^{(n)} = x_m^{(n)} - K^T\hat{e} + u_c + [f(x,t) - \hat{f}(\hat{x},t)] + \\ +[g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\end{gathered}$$

It holds $e = x - x_m \Rightarrow x^{(n)} = e^{(n)} + x_m^{(n)}$. Substituting $x^{(n)}$ in the above equation gives

$$\begin{gathered}\dot{e} = Ae - BK^T\hat{e} + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + \\ +[g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\} \\ e_1 = C^T e\end{gathered} \tag{30}$$

$\hat{e} = \left(\hat{e}, \dot{\hat{e}}, \ddot{\hat{e}}, \cdots, \hat{e}^{(n-1)}\right)^T$ and A, C, K are given by Eq. (18). According to Eq. (30) the observer is:

$$\begin{gathered}\dot{\hat{e}} = A\hat{e} - BK^T\hat{e} + K_o[e_1 - C^T\hat{e}] \\ \hat{e}_1 = C^T\hat{e}\end{gathered} \tag{31}$$

The observation gain $K_o = [k_{o_0}, k_{o_1}, \cdots, k_{o_{n-2}}, k_{o_{n-1}}]$ is selected so as to assure the convergence of the observer. Subtracting Eq. (31) from Eq. (30) one gets

$$\dot{\tilde{e}} = (A - K_o C^T)\tilde{e} + Bu_c + B\{[f(x,t) - \hat{f}(\hat{x},t)] + \\ +[g(x,t) - \hat{g}(\hat{x},t)]u + \tilde{d}\}$$

$$\tilde{e}_1 = C\tilde{e} \tag{32}$$

The additional term u_c which appeared in Eq. (13) is also introduced in the observer-based control of the DC-motor to compensate for: i) The external disturbances $\tilde{d}$, ii) The state vector estimation error $\tilde{e} = e - \hat{e} = x - \hat{x}$, iii) The approximation error of the nonlinear functions $f(x,t)$ and $g(x,t)$, denoted as $w = [f(x,t) - \hat{f}(\hat{x},t)] + [g(x,t) - \hat{g}(\hat{x},t)]u$. The control u_c consists of: i) the H_∞ control term u_a, for the compensation of d and w, ii) the control term u_b, for the compensation of the observation error $\tilde{e}$.The control scheme is depicted in Fig. 4.

$$\begin{array}{c} u_a = -\frac{1}{r}B^T P\tilde{e} \\ u_b = -K_o^T P_1 \hat{e} \end{array} \tag{33}$$

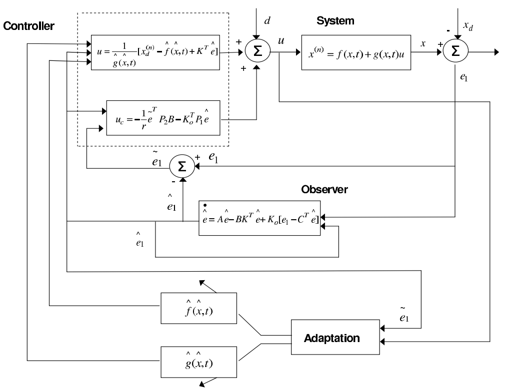

Fig. 4 The proposed H_∞ control scheme in the case of output feedback

5.2 Approximation of $f(x,t)$ and $g(x,t)$ using output feedback

The approximation of functions $f(\hat{x},t)$ and $g(\hat{x},t)$ of Eq. (29) can be carried out again with Takagi-Sugeno neuro-fuzzy networks of zero or first order (see again Fig. 2). These consist of rules of the form:

R^l : IF $\hat{x}$ is A_1^l AND $\dot{\hat{x}}$ is A_2^l AND $\cdots$ AND $\hat{x}^{(n-1)}$ is A_n^l THEN $\bar{y}^l = \sum_{i=1}^n w_i^l \hat{x}_i + b^l, \;\; l = 1, 2, \cdots, L$

The output of the Takagi-Sugeno model is calculated by taking the average of the consequent part of the rules

$$\hat{y} = \frac{\sum_{l=1}^L \bar{y}^l \prod_{i=1}^n \mu_{A_i}^l(\hat{x}_i)}{\sum_{l=1}^L \prod_{i=1}^n \mu_{A_i}^l(\hat{x}_i)}$$

where $\mu_{A_i^l}$ is the membership function of x_i in the fuzzy set A_i^l. The training of the neuro-fuzzy networks is carried out with 1^{st} order gradient algorithms, in pattern mode, i.e. by processing only one data pair (x_i, y_i) at every time step i. The estimation of $f(x,t)$ and $g(x,t)$ can be written as

$$\hat{f}(\hat{x}|\theta_f) = \theta_f^T \phi(\hat{x}) \; \hat{g}(\hat{x}|\theta_g) = \theta_g^T \phi(\hat{x}) \tag{34}$$

where $\phi(\hat{x})$ are kernel functions with elements

$$\phi^l(\hat{x}) = \frac{\prod_{i=1}^n \mu_{A_i}^l(\hat{x}_i)}{\sum_{l=1}^L \prod_{i=1}^n \mu_{A_i}^l(\hat{x}_i)} \;\; l = 1, 2, \cdots, L$$

It is assumed that that the weights θ_f and θ_g vary in the bounded areas M_{θ_f} and M_{θ_g}, while x and $\hat{x}$ remain in the bounded areas U_x and $U_{\hat{x}}$ respectively. The values of θ_f and θ_g for optimal approximation are:

$$\begin{array}{c} \theta_f^* = arg \;\; min_{\theta_f \in M_{\theta_f}} [sup_{x \in U_x, \hat{x} \in U_{\hat{x}}} |f(x) - \hat{f}(\hat{x}|\theta_f)|], \\ \theta_g^* = arg \;\; min_{\theta_g \in M_{\theta_g}} [sup_{x \in U_x, \hat{x} \in U_{\hat{x}}} |g(x) - \hat{g}(\hat{x}|\theta_g)|]. \end{array}$$

The approximation error of $f(x,t)$ and $g(x,t)$ is given by

$$\begin{array}{c} w = [\hat{f}(\hat{x}|\theta_f^*) - f(x,t)] + [\hat{g}(\hat{x}|\theta_g^*) - g(x,t)]u \Rightarrow \\ w = \{[\hat{f}(\hat{x}|\theta_f^*) - f(x|\theta_f^*)] + [f(x|\theta_f^*) - f(x,t)]\} + \\ +\{[\hat{g}(\hat{x}|\theta_g^*) - g(\hat{x}|\theta_g^*)] + [g(\hat{x}|\theta_g^*) g(x,t)]u\} \end{array}$$

where, i) $\hat{f}(\hat{x}|\theta_f^*)$ is the approximation of f for the best estimation θ_f^* of the weights' vector θ_f, ii) $\hat{g}(\hat{x}|\theta_g^*)$ is the approximation of g for the best estimation θ_g^* of the weights' vector θ_g. The approximation error w can be decomposed into w_a and w_b, where

$$\begin{array}{c} w_a = [\hat{f}(\hat{x}|\theta_f) - \hat{f}(\hat{x}|\theta_f^*)] + [\hat{g}(\hat{x}|\theta_g) - \hat{g}(\hat{x}|\theta_g^*)]u, \\ w_b = [\hat{f}(\hat{x}|\theta_f^*) - f(x,t)] + [\hat{g}(\hat{x}|\theta_g^*) - g(x,t)]u. \end{array}$$

Finally, the following two parameters are defined: $\tilde{\theta}_f = \theta_f - \theta_f^*$ and $\tilde{\theta}_g = \theta_g - \theta_g^*$.

6 Lyapunov stability analysis of the control loop in the case of output feedback

The adaptation law of the neuro-fuzzy approximators weights θ_f and θ_g as well as of the supervisory control term u_c are derived from the requirement for negative definiteness of the Lyapunov function

$$V=\frac{1}{2}\hat{e}^TP_1\hat{e}+\frac{1}{2}\tilde{e}^TP_2\tilde{e}+\frac{1}{2\gamma_1}\tilde{\theta}_f^T\tilde{\theta}_f+\frac{1}{2\gamma_2}\tilde{\theta}_g^T\tilde{\theta}_g \tag{35}$$

The selection of the Lyapunov function is based on the following two principles of indirect adaptive control: i) $\hat{e}:\lim_{t\to\infty}\hat{x}(t)=x_d(t)$, ii) $\tilde{e}:\lim_{t\to\infty}\hat{x}(t)=x(t)$ which yields $lim_{t\to\infty}x(t)=x_d(t)$. Substituting Eq. (30), and Eq. (32), into Eq. (35) and differentiating results into

$$\begin{gathered}\dot{V}=\frac{1}{2}\hat{e}^T(A-BK^T)^TP_1\hat{e}+\frac{1}{2}\tilde{e}^TCK_o^TP_1\hat{e}+\\+\frac{1}{2}\hat{e}^TP_1(A-BK^T)\hat{e}++\frac{1}{2}\hat{e}^TP_1K_oC^T\tilde{e}+\\+\frac{1}{2}\tilde{e}^T(A-K_oC^T)^TP_2\tilde{e}+\frac{1}{2}B^TP_2\tilde{e}(u_c+w+d)+\\+\frac{1}{2}\tilde{e}^TP_2(A-K_oC^T)\tilde{e}+\frac{1}{2}\tilde{e}^TP_2B(u_c+w+d)+\\\frac{1}{\gamma_1}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f+\frac{1}{\gamma_2}\tilde{\theta}_g^T\dot{\tilde{\theta}}_g\end{gathered}$$

Assumption 1: For given positive definite matrices Q_1 and Q_2 there exist positive definite matrices P_1 and P_2, which are the solution of the following Riccati equations

$$(A-BK^T)^TP_1+P_1(A-BK^T)+Q_1=0$$

$$\begin{gathered}(A-K_oC^T)^TP_2+P_2(A-K_oC^T)-\\-P_2B(\frac{2}{r}-\frac{1}{\rho^2})B^TP_2+Q_2=0\end{gathered}$$

$$P_2B=C \tag{36}$$

The conditions given in Eq. (36) are related to the requirement that the systems described by Eq. (31) and Eq. (32), are strictly positive real. Substituting Eq. (36) into $\dot{V}$ yields

$$\begin{gathered}\dot{V}=-\frac{1}{2}\hat{e}^TQ_1\hat{e}+\tilde{e}^TCK_o^TP_1\hat{e}-\\-\frac{1}{2}\tilde{e}^T\{Q_2-P_2B(\frac{2}{r}-\frac{1}{\rho^2})B^TP_2\}\tilde{e}+\\+B^TP_2\tilde{e}(u_c+w+d)+\frac{1}{\gamma_1}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f+\frac{1}{\gamma_2}\tilde{\theta}_g^T\dot{\tilde{\theta}}_g\end{gathered} \tag{37}$$

Substituting u_a and u_b in $\dot{V}$ and assuming that Eq. (36) holds, after some operations one gets

$$\begin{gathered}\dot{V}=-\frac{1}{2}\hat{e}^TQ_1\hat{e}-\frac{1}{2}\tilde{e}^TQ_2\tilde{e}-\\-\frac{1}{2\rho^2}\tilde{e}^TP_2BB^TP_2\tilde{e}+B^TP_2\tilde{e}(w+d)+\\+\frac{1}{\gamma_1}\tilde{\theta}_f^T\dot{\tilde{\theta}}_f+\frac{1}{\gamma_2}\tilde{\theta}_g^T\dot{\tilde{\theta}}_g\end{gathered} \tag{38}$$

It holds that

$$\begin{gathered}\dot{\tilde{\theta}}_f=\dot{\theta}_f-\dot{\theta}_f^*=\dot{\theta}_f\\\dot{\tilde{\theta}}_g=\dot{\theta}_g-\dot{\theta}_g^*=\dot{\theta}_g\end{gathered}$$

The following weight adaptation laws are considered [10]:

$$\dot{\theta}_f=\{\begin{matrix}-\gamma_1\tilde{e}^TP_2B\phi(\hat{x})\ if\ ||\theta_f||\in M_{\theta_f}\\0\ ||\theta_f||\notin M_{\theta_f}\end{matrix} \tag{39}$$

$$\dot{\theta}_g=\{\begin{matrix}-\gamma_2\tilde{e}^TP_2B\phi(\hat{x})u_c\ if\ ||\theta_g||\in M_{\theta_g}\\0\ ||\theta_g||\notin M_{\theta_g}\end{matrix} \tag{40}$$

Substituting Eq. (39) and using Eq. (34) and (39) results into

$$\begin{gathered}\dot{V}=-\frac{1}{2}\hat{e}^TQ_1\hat{e}-\frac{1}{2}\tilde{e}^TQ_2\tilde{e}-\\\frac{1}{2\rho^2}\tilde{e}^TP_2BB^TP_2\tilde{e}+\tilde{e}^TP_2B(w+d)-\\-\tilde{e}^TP_2B\{[\hat{f}(\hat{x}|\theta_f)+\hat{g}(\hat{x}|\theta_f)u]-\\-[\hat{f}(\hat{x}|\theta_f^*)+\hat{g}(\hat{x}|\theta_g^*)u]\}\end{gathered} \tag{41}$$

where

$$[\hat{f}(\hat{x}|\theta_f)+\hat{g}(\hat{x}|\theta_f)u]-[\hat{f}(\hat{x}|\theta_f^*)+\hat{g}(\hat{x}|\theta_g^*)u]=w_a.$$

Thus setting $w_1=w+w_a+d$ one finally gets

$$\begin{gathered}\dot{V}=-\frac{1}{2}\hat{e}^TQ_1\hat{e}\frac{1}{2}\tilde{e}^TQ_2\tilde{e}-\frac{1}{2\rho^2}\tilde{e}^TP_2BB^TP_2\tilde{e}+\\+\frac{1}{2}w_1^TB^TP_2\tilde{e}+\frac{1}{2}\tilde{e}^TP_2Bw_1\end{gathered}$$

Lemma: The following inequality holds

$$\begin{gathered}\frac{1}{2}\tilde{e}^TP_2Bw_1+\frac{1}{2}w_1^TB^TP_2\tilde{e}-\frac{1}{2\rho^2}\tilde{e}^TP_2BB^TP_2\tilde{e}\\\leq\frac{1}{2}\rho^2w_1^Tw_1\end{gathered} \tag{42}$$

Proof: The binomial $(\rho a-\frac{1}{\rho}b)^2\geq 0$ is considered. Expanding the left part of the above inequality results in:

$$ab-\frac{1}{2\rho^2}b^2\leq\frac{1}{2}\rho^2a^2\Rightarrow\frac{1}{2}ab+\frac{1}{2}ab-\frac{1}{2\rho^2}b^2\leq\frac{1}{2}\rho^2a^2 \tag{43}$$

Substituting $a=w_1$ and $b=\tilde{e}^TP_2B$ and the previous relation one gets Eq. (42) $\diamond$.

Eq. (42) is used in $\dot{V}$, and the right part of the associated inequality is enforced

$$\dot{V}\leq-\frac{1}{2}\hat{e}^TQ_1\hat{e}-\frac{1}{2}\tilde{e}^TQ_2\tilde{e}+\frac{1}{2}\rho^2w_1^Tw_1 \tag{44}$$

Hence, the H_∞ performance criterion of Eq. (12) is derived. For ρ sufficiently small Eq. (44) will be true and the H_∞ tracking criterion will be satisfied. In that case, the integration of $\dot{V}$ from 0 to T gives

$$2V(T) + \int_0^T ||E||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||w_1||^2 dt \tag{45}$$

where $E = [\hat{e}, \tilde{e}]^T$ and $Q = diag[Q_1, Q_2]^T$. It is assumed that there exists a positive constant $M_w > 0$ such that $\int_0^\infty ||w_1||^2 dt \leq M_w$. Therefore for the integral $\int_0^T ||E||_Q^2 dt$ one gets

$$\int_0^\infty ||E||_Q^2 dt \leq 2V(0) + \rho^2 M_w \tag{46}$$

Thus,the integral $\int_0^\infty ||E||_Q^2 dt$ is bounded and according to Barbalat's Lemma

$$\begin{array}{c} lim_{t\to\infty} E(t) = 0 \Rightarrow lim_{t\to\infty} \hat{e}(t) = 0 \\ lim_{t\to\infty} \tilde{e}(t) = 0 \end{array} \tag{47}$$

Therefore $lim_{t\to\infty} e(t) = 0$.

7 Simulation tests

7.1 Performance of the state feedback controller

The performance of the state feedback controller was tested in the tracking of several reference trajectories. The time step of the simulation experiments was taken to be $T_s = 0.01$ sec.

For $r = 1.0$ and $\rho = 1.0$ the Riccati equation given in Eq. (23) was solved. The basis functions used in the estimation of $f(x,t)$ and $g(x,t)$ were $\mu_{A_j}(\hat{x}) = e^{(\frac{\hat{x}-c_j}{\sigma})^2}, j = 1, \cdots, 3$. In the associated fuzzy rule base there are three inputs $x_1 = \theta$, $\dot{x}_1 = \dot{\theta}$ and $\ddot{x}_1 = \ddot{\theta}$. The universe of discourse of each input variable consisted of 3 fuzzy sets. Consequently 27 fuzzy rules were derived which had the following form:

$$\begin{array}{c} R^l : IF \ x_1 \ is \ A_1^l \ AND \ \dot{x}_1 \ is \ A_2^l \\ AND \ \ddot{x}_1 \ is \ A_3^l \ THEN \ \hat{f}^l \ is \ b^l \end{array} \tag{48}$$

and the approximation of function $f(x,t)$ in the motor's model of Eq. (9) was given by

$$\hat{f}(x,t) = \frac{\sum_{l=1}^{27} \hat{f}^l \prod_{i=1}^{3} \mu_{A_i}^l(x_i)}{\sum_{l=1}^{27} \prod_{i=1}^{3} \mu_{A_i}^l(x_i)} \tag{49}$$

The centers $c_i^{(l)}$, $i = 1, \cdots, 3$ can take values in the set $\{-1.0, 0.0, 1.0\}$ while the variances $v_i^{(l)}$, $i = 1, \cdots, 3$ were given the value $v_i^{(l)} = 2.2$. Thus taking the possible combinations the following R^l, $l = 1, \cdots, 27$ are derived where the associated centers and variances are defined as:

Rule	$c_1^{(l)}$	$c_2^{(l)}$	$c_3^{(l)}$	$v^{(l)}$
$R^{(1)}$	-1.0	-1.0	-1.0	2.2
$R^{(2)}$	-1.0	-1.0	0.0	2.2
$R^{(3)}$	-1.0	-1.0	1.0	2.2
$R^{(4)}$	-1.0	0.0	-1.0	2.2
$R^{(5)}$	-1.0	0.0	0.0	2.2
$R^{(6)}$	-1.0	0.0	1.0	2.2
...	...	...	...	...
...	...	...	...	...
$R^{(27)}$	1.0	1.0	1	1

Similar was the fuzzy rule based that was used in the approximation of function $g(x,t)$ of Eq. (9). The learning rates γ_1 and γ_2 of the neurofuzzy networks were suitably tuned. The controller's gain $K = [k_0, k_1, k_0]^T$ was suitably selected so as to result in a Hurwitz stable polynomial and to assure the asymptotic convergence of the tracking error to zero. In the first half of the simulation time the training of the neuro-fuzzy approximators was carried out. In the second half, the estimated functions $\hat{f}(x,t)$ and $\hat{g}(x,t)$ were used to derive the control signal. First the performance of the proposed state feedback controller was tested in the tracking a sinusoidal set-point.

- The position and velocity variations for a sinuoidal set-point are depicted in Fig. 5 and Fig. 6, respectively.
- The acceleration tracking succeeded for the sinusoidal set-point is shown in Fig. 7, while associated control input is shown in Fig. 8

From the simulation tests the following remarks can be made: (i) adaptive fuzzy H_∞ control based on state feedback succeeds excellent tracking of the reference motor's angle θ_d. Overshooting depends on the selection of the feedback gain K, (ii) Excellent tracking of the reference angular velocity $\dot{\theta}_d$ is also achieved, (iii) The variation of the control input (field voltage) is smooth. This was due to the proper selection of the feedback gain K, (iv) The neuro-fuzzy networks can succeed good approximations of the unknown functions $f(x,t)$ and $g(x,t)$. The accuracy in the estimation of $g(x,t)$ is important for the convergence of the control algorithm.

7.2 Performance of the output feedback controller

The performance of the output feedback controller was also tested in the tracking of several set-points. The time step was again taken to be $T_s = 0.01$ sec.

The controller's feedback gain $K = [k_0, k_1, k_2]^T$ and the observer's gain $K_o = [k_{o_0}, k_{o_1}, k_{o_2}]^T$ were suitably selected so as to assure the asymptotic elimination of the tracking and observation errors respectively. The basis functions used in the estimation of $f(x,t)$ and $g(x,t)$ were $\mu_{A_j}(\hat{x}) = e^{(\frac{\hat{x}-c_j}{\sigma})^2}, j = 1, \cdots, 3$. Since there were three inputs $\hat{x}_1$, $\dot{\hat{x}}_1$ and $\ddot{\hat{x}}_1$ and the associated

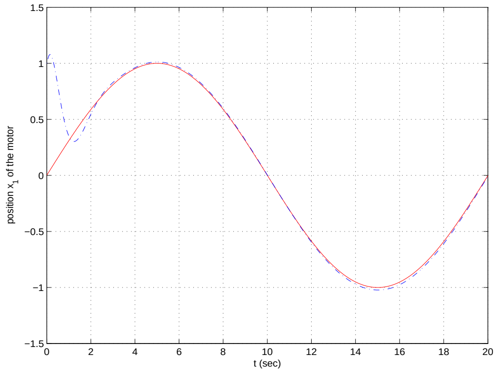

Fig. 5 Full state feed-back control: state x_1 (dashed line) tracks a sinusoidal set-point (continuous line)

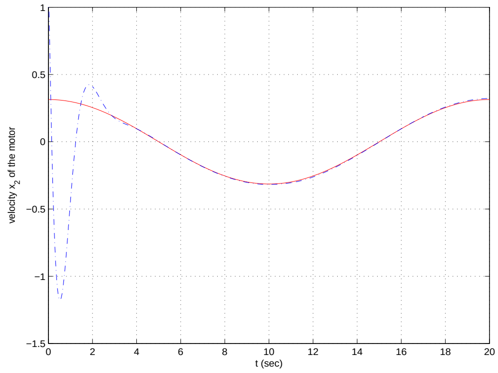

Fig. 6 Full state feed-back control: state x_2 (dashed line) tracks a sinusoidal set-point (continuous line)

universes of discourse consisted of 3 fuzzy sets there were again 27 fuzzy rules of the form:

$$\begin{array}{l} R^l : IF\ \hat{x}_1\ is\ A_1^l\ AND\ \hat{\dot{x}}_1\ is\ A_2^l \\ AND\ \hat{\ddot{x}}_1\ is\ A_3^l\ THEN\ \hat{f}^l\ is\ b^l \end{array} \tag{50}$$

where the approximation of function $f(x,t)$ is given by

$$\hat{f}(\hat{x},t) = \frac{\sum_{l=1}^{27} \hat{f}^l \prod_{i=1}^{2} \mu_{A_i}^l(\hat{x}_i)}{\sum_{l=1}^{27} \prod_{i=1}^{2} \mu_{A_i}^l(\hat{x}_i)}. \tag{51}$$

The centers $c_i^l,\ i=1,\cdots,3$ take values from the set $\{-1.0, 0.0, 1.0\}$ while the variance of the fuzzy sets $v_i^l,\ i=1,\cdots,3$ is given again the value 1. Thus, the centers $c_i^{(l)},\ i=1,2,3$ and the variances $v^{(l)}$ of each rule are as follows

Rule	$c_1^{(l)}$	$c_2^{(l)}$	$c_3^{(l)}$	$v^{(l)}$
$R^{(1)}$	-1.0	-1.0	-1.0	2.2
$R^{(2)}$	-1.0	-1.0	0.0	2.2
$R^{(3)}$	-1.0	-1.0	1.0	2.2
$R^{(4)}$	-1.0	-0.0	-1.0	2.2
$R^{(5)}$	-1.0	-0.0	0.0	2.2
$R^{(6)}$	-1.0	-0.0	1.0	2.2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$R^{(27)}$	1.0	1.0	1.0	2.2

Similar was the fuzzy rule base that provided the approximation of function $g(x,t)$ of Eq. (9). The first half of the simulation time was used for training the neuro-fuzzy approximators and a measurable state vector was used. Matrices P_1 and P_2 were obtained from the solution of the Riccati equation given in Eq. (36). First, the proposed controller was used for tracking a sinusoidal set-point:

- The position and velocity tracking succeeded in the case of the sinusoidal set-point are depicted in Fig. 9 and Fig. 10, respectively.
- The acceleration tracking succeeded for the sinusoidal set-point is shown in Fig. 11, while the associated control input is shown in Fig. 12

Finally the estimation succeeded for the state $x_1 = \theta$ of the motor in the case of the sinusoidal set-point is given in Fig. 13.

Adaptive fuzzy H_∞ control with output feedback has the same advantages as state-feedback based adaptive fuzzy control. These are summarized in the following: (i) removal of any dependence upon identification of the mathematical model expressing the dynamics of the motor, (ii) since training of the neurofuzzy approximators contained in the adaptive fuzzy H_∞ controller is repeatedly undertaken in every control cycle, any changes to the motor dynamics can be identified on-line, and hence the control strategy is useful for time varying motor models, (iii) regarding operation under external

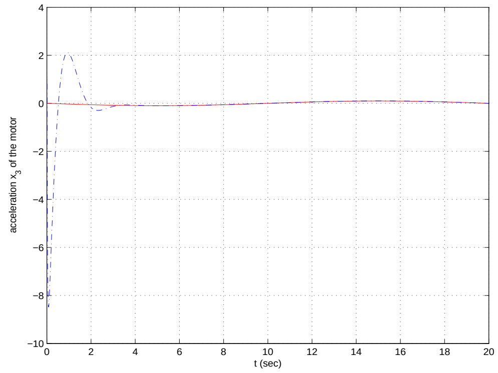

Fig. 7 Full state feed-back control: (a) state x_3 (dashed line) tracks a sinusoidal set-point (continuous line)

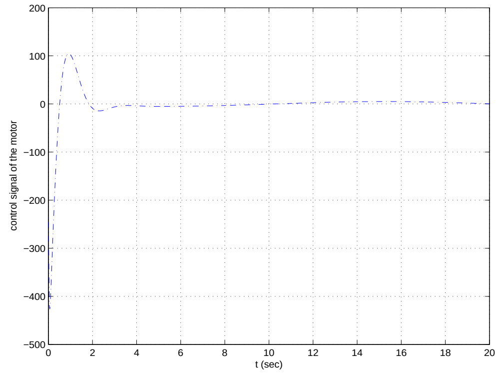

Fig. 8 Full state feed-back control: (a) state x_3 (dashed line) tracks a sinusoidal set-point (continuous line) (b) control signal (dashed line) for the dc-motor

disturbances and measurement noise, robustness of the closed loop is succeeded. Moreover, it should be noted that in the case of adaptive fuzzy control with output feedback there is no need to use additional sensors to measure the velocity and the acceleration of the motor, since the state vector is reconstructed with the use of an observer.

8 Conclusions

An adaptive fuzzy control method for DC motors was developed. Neuro-fuzzy networks were used to approximate the unknown system dynamics, while the control signal was generated either using full state-vector feedback or only feedback of the systems output. With the proposed control scheme, the speed and the position of the rotor shaft were forced to follow any arbitrary selected trajectory under variable load torque. The method is suitable for any servo-system application employing any type of motor.

The key ideas in the design of the proposed controller are (i) to transform the nonlinear control problem into a regulation problem through suitable output feedback, (ii) to design neuro-fuzzy approximators that receive as inputs the parameters of reconstructed state vector and give as output an estimation of the system's unknown dynamics, (iii) to use an H_∞ control term for the compensation of external disturbances and modelling errors. (iv) to use Lyapunov stability analysis in order to find the learning law for the neuro-fuzzy approximators, and a supervisory control term for disturbance and modelling error rejection. In case that the motor's state vector is not completely measurable the inclusion of a state observer is the control loop enables control using only output feedback.

The control method presented in this paper is generic and suitable for SISO nonlinear systems with parametric uncertainty or subject to external disturbances. The model of the DC-motor is a case study that demonstrates the usefulness of the proposed method. The novelties in the control of uncertain dynamical systems, that is shown in this paper, can be summarized in the following: (i) No prior knowledge of the system's dynamical model is required. The control proposed is a model-free approach. The unknown parts of the system's dynamics are approximated with the use of neuro-fuzzy networks (ii) No complete knowledge of the system's state vector is required to design a feedback control law. The state vector is reconstructed with the use of a state observer, (iii) The proposed control approach can compensate for changes which occur online to the system's dynamics, (iv) The proposed control method provides improved robustness to measurement noise and external disturbances.

It has to be noted that the design of the output feedback-based adaptive fuzzy controller is more complicated. This is because the existence of this controller depends on whether two Riccati equations can be solved simultaneously. Finally, regarding the performance of the two proposed controllers (output and state-feedback

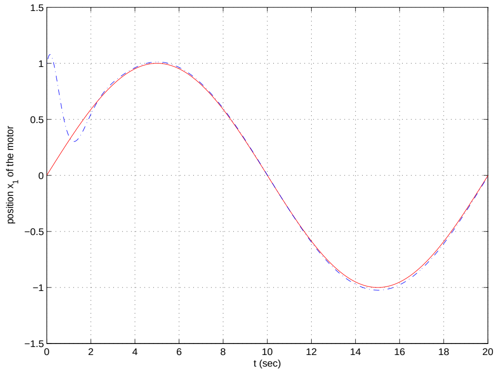

Fig. 9 Control using output feed-back: state x_1 (dashed line) tracks a sinusoidal set-point (continuous line)

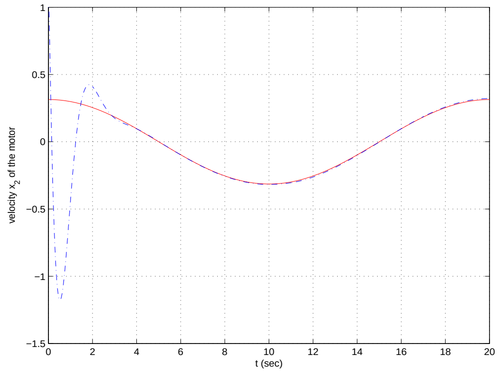

Fig. 10 Control using output feed-back: state x_2 (dashed line) tracks a sinusoidal set-point (continuous line)

based) no remarkable differences were recorded in the simulation experiments. Both approaches showed robustness to modeling errors and external disturbances of the DC-motor model and H_∞ tracking performance was succeeded.

9 References

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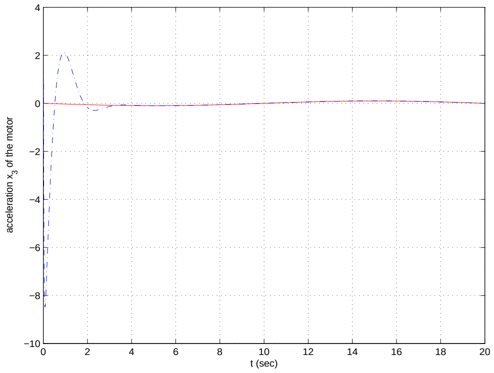

Fig. 11 Control using output feed-back: state x_3 (dashed line) tracks a sinusoidal set-point (continuous line)

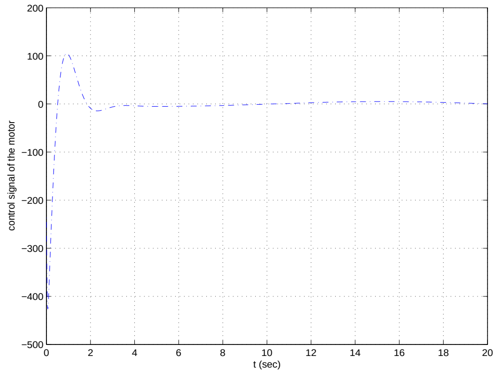

Fig. 12 Control using output feed-back: control signal (dashed line) for the DC motor

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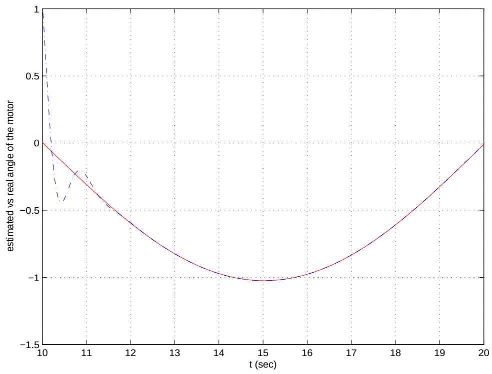

Fig. 13 Estimated (dashed line) vs real value (continuous line) of the angle θ of the motor in the case of sinusoidal set-point

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