

# MULTI-MODEL ADAPTIVE CONTROL

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## Abstract

A multiple models adaptive control system will be presented. The advantages of this control with respect to the classical control will be illustrated on a level control system with nonlinear model plant. A recent recursive method in open and closed-loop identification and an R-S-T controller design has been proposed to guarantee the performances in the adaptive control scheme. The real time control system implementation confirms the opportunity of using the multi-models adaptive control architecture in the case when the nonlinear plant model introduces a typical large parameter variation. The article presents a method to choose the algorithms, in order to meet the adaptive regulation performance requirements. The obtained results show that the control strategy based on a single model and on a single controller generates a time response that is more affected by noise than the response given by an adaptive strategy. The multiple models adaptive control procedure proposed has the following advantages: a more precise model is chosen for the closed loop operating system, the R-S-T adaptive control ensures very good real time results for closed loop nonlinear system. It can be appreciate that the multiple models adaptive control can be recommended to improve the performances of the nonlinear control systems.

**Keywords: multi-models, closed-loop identification, R-S-T control, adaptive control, real time application.**

## Presenting Author’s biography

Cristian Flutur is a teaching assistant at the Automatic Control and Computer Science Faculty. His teaching areas regard Signals and Systems, Feedback Systems Theory and Optimization Techniques. He also carries out research activities and handles research projects within the ACPC research center. He graduated the same Faculty in September 2006. He completed his diploma project – “Nonlinear Models’ Sensitivity in the Context of Parametric Incertitude” – at Ecole Centrale de Lille, in France.



## 1 Introduction

Since 90's years different approaches of multi-model control have been developed. The Balakrishnan's and Narendra's first papers [1], which proposed several stability and robustness methods using classical switching and tuning algorithms, have to be mentioned. Later, the research in this field determined the extension and the improvement of multi-model control concept.

Magill and Lainiotis introduced the model representation through Kalman filters [2]. In order to maintain the stability of minimum phase systems, Middleton improved the switching procedure using an algorithm with hysteresis. Petridis', Kehagias' and Toscano's work was focused on nonlinear systems with time variable. Landau and Karimi have some contributions regarding the use of several particular parameter adaptation procedures, namely CLOE (Closed Loop Output Error) [3]. The multi-model control version proposed by Narendra is based on neural networks. Finally, Dubois, Dieulat and Borne apply fuzzy procedures for switching and sliding mode control.

We propose a multi-model control procedure with closed loop identification for model parameter re-estimation and with adaptive control design after each switching operation.

Next, this paper emphasizes a new procedure for the multi-model control systems design which leads to quality improvement of the real time nonlinear control systems.

Consider the set of the models:

$$\mathbf{M} = \{M_1, M_2, M_3 \dots M_n\}$$

and the class of correspondent controllers:

$$\mathbf{C} = \{C_1, C_2, C_3 \dots C_n\},$$

integrated in the closed-loop configuration, as in Fig. 1.

The input and output of the plant  $\mathbf{P}$  are  $\mathbf{u}$  and  $\mathbf{y}$  respectively.

The  $M_i$  ( $i=1,2,\dots, n$ ) models are a priori evaluated. For each model  $M_i$  we are designing the controller  $C_i$  so that the pair  $(M_i, C_i)$  ensures the nominal performances.

The main idea of the multi-model adaptive control consists in choosing the best model included in  $\mathbf{M}$  to apply the correspondent controller and continuing in the adaptive way towards current operating point of the plant.

In order to use this mechanism the identification problem is developed in two steps:

- The model with smallest error with respect to a performance criterion is chosen (switching - step). After this operation the correspondent control input  $\mathbf{u}$  is attached to the chosen model.
- Using the adaptive strategy for real time control system, the parameters of the model are adjusted and the new control algorithm is computed (tuning - step).

## 2 Choice of the model

The model-error at the  $k$  instant is defined as the difference between the output  $y_i$  of the model  $M_i$  and the output  $\mathbf{y}$  of the plant:

$$\varepsilon_i(k) = y(k) - y_i(k) \quad (1)$$

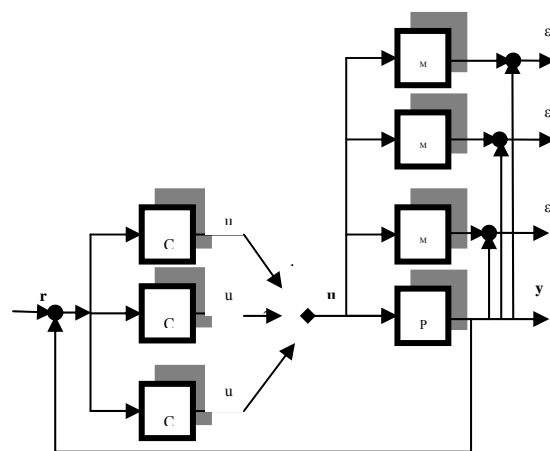


Fig. 1. Multimodel structure

The performance criterion which is used as the selection rule is defined below:

$$J_i(k) = \alpha \varepsilon_i^2(k) + \beta \sum_{j=1}^k e^{-\lambda(k-j)} \varepsilon_i^2(j) \quad (2)$$

where  $\alpha > 0$  and  $\beta > 0$  are the weighting factors on the instantaneous measures and the long term accuracy;  $\lambda > 0$  is the forgetting factor.

The choosing of the  $\alpha$ ,  $\beta$  and  $\lambda$  parameters depends of the plant:

- $\alpha = 1$  and  $\beta = 0 \rightarrow$  for the fast systems (good performances with respect to parameters changes, sensitive to disturbance);
- $\alpha = 0$  and  $\lambda = 0 \rightarrow$  for the slow systems (bad performances with respect to parameters changes, good performances with respect to disturbance).

### 3 Closed-loop recursive identification

A closed-loop adaptive method (filtered closed loop error- FCLOE identification) for the adjustable predictor is considered. This method computes the parameters of the model in order to minimize the closed loop output prediction error  $\varepsilon_{CL}$  using the filtered data  $u$  and  $y$  [4]. A FCLOE identification scheme is presented in Fig.2.

The basic idea is to substitute (by filtering of  $u$  and  $y$ ) the prediction error  $\varepsilon_{LS}$  with closed-loop output error  $\varepsilon_{CL}$ . The filter depends of the control algorithm.

The FCLOE – algorithm in least squares recursive form is the following:

$$\theta(k+1) = \theta(k) + F(k)\phi_f(k)\varepsilon_{LS}(k+1) \quad (3)$$

$$F(k+1) = F(k) - \frac{F(k)\phi_f(k)\phi_f(k)^T F(k)}{1 + \phi_f(k)^T F(k)\phi_f(k)}$$

$$F(0) = \alpha I, \alpha > 0 \quad (4)$$

$$\varepsilon_{CL}(k+1) = \frac{y(k+1) - \theta^T(k)\phi_f(k)}{1 + \phi_f(k)^T F(k)\phi_f(k)} \quad (5)$$

where ,

$\theta(k)$  is the parameter vector;

$\phi_f(k)$  is the filtered observation vector;

$F(k)$  is the gain adaptation matrix;

$\varepsilon_{CL}$  is the closed-loop prediction error.

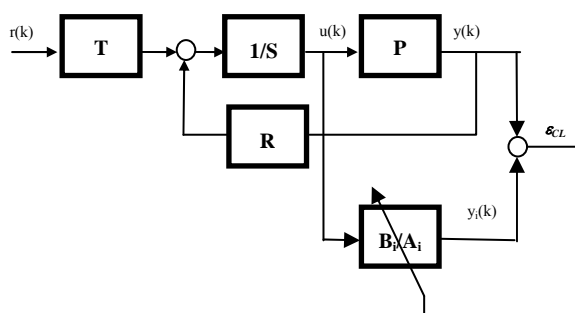


Fig. 2. Close loop identification scheme

### 4 Model based control (re)design

For the  $M_i$  model we design a controller  $C_i$  that satisfies the desired nominal performances. The RST polynomial algorithm with two degrees of liberty, for  $C_i$  controller is proposed (see Fig. 3):

In this case the input  $u(k)$  is:

$$u(k) = \frac{T(q^{-1})}{S(q^{-1})}r(k) - \frac{R(q^{-1})}{S(q^{-1})}y(k) \quad (6)$$

The disturbance rejection is ensured by the  $R(q^{-1})$ ,  $S(q^{-1})$  polynomials, obtained solving the equation:

$$P_C(q^{-1}) = A(q^{-1})S(q^{-1}) + B(q^{-1})R(q^{-1}) \quad (7)$$

where,

pair  $(A(q^{-1}), B(q^{-1}))$  represents the plant model;

$P_C(q^{-1})$  is the closed-loop characteristic polynomial.

The reference tracking performance is ensured by the choice of the  $T(q^{-1})$  polynomial. For each model  $(A_i, B_i)$  a  $C_i$  control algorithm ( $R_i, S_i, T_i$  polynomials) will be computed respectively. The adaptive pole placement method is used for achieved performances in closed-loop.

There are two possibilities for the adaptive design approach:

A. Disturbance rejection adaptive algorithm:

1. Re-identification of the model  $M_{k+1}$  using the relation (3), where the filtered data is

$$\Phi_f(k) = \frac{S}{P_C} \Phi(k).$$

$$M_{k+1} = \frac{B_{k+1}(q^{-1})}{A_{k+1}(q^{-1})} \quad (8)$$

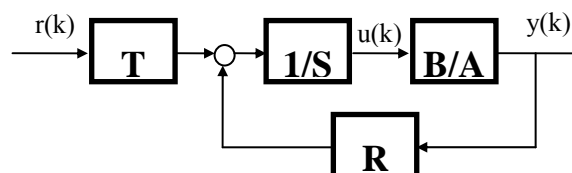


Fig. 3. RST control algorithm

2. Evaluation of the pair  $R_{k+1}(q^{-1})$ ,  $S_{k+1}(q^{-1})$  from equation:

$$P_C(q^{-1}) = A_{k+1}(q^{-1})S(q^{-1}) + B_{k+1}(q^{-1})R(q^{-1}) \quad (9)$$

3. Computation of the input  $u(k+1)$ :

$$u(k+1) = \frac{T(q^{-1})}{S_{k+1}(q^{-1})}r(k) - \frac{R_{k+1}(q^{-1})}{S_{k+1}(q^{-1})}y(k) \quad (10)$$

B. Reference tracking adaptive algorithm:

1. Identification of the model  $M_{k+1}$ :

$$M_{k+1} = \frac{B_{k+1}(q^{-1})}{A_{k+1}(q^{-1})} \quad (11)$$

2. Computation of  $P_{C_{k+1}}(q^{-1})$  using the equation:

$$P_{Ck+1}(q^{-1}) = A_{k+1}(q^{-1})S(q^{-1}) + B_{k+1}(q^{-1})R(q^{-1}) \quad (12)$$

3. Computation  $T_{k+1}(q^{-1})$  with relation:

$$T_{k+1}(q^{-1}) = \frac{P_{k+1}(1)}{B_{k+1}(1)} P_{Ck+1}(q^{-1}) \quad (13)$$

4. Computation of the input  $u(k+1)$

$$u(k+1) = \frac{T_{k+1}(q^{-1})}{S(q^{-1})} r(k) - \frac{R(q^{-1})}{S(q^{-1})} y(k) \quad (14)$$

The main experimental results from real time multi-models adaptive control system will be presented below.

## 5 Experimental results

We have evaluated the achieved performances of the adaptive multi-model control using an experimental installation as in Fig. 4. The main goal is to control in closed loop the level in Tank 1. There is a nonlinear relation between the level  $L$  and the flow  $F$ .

$$F = a\sqrt{2gL} \quad (15)$$

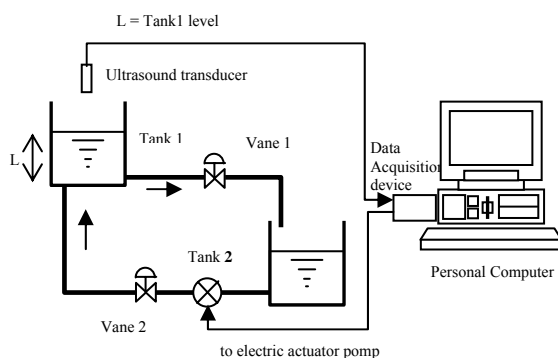


Fig. 4. Experimental installation

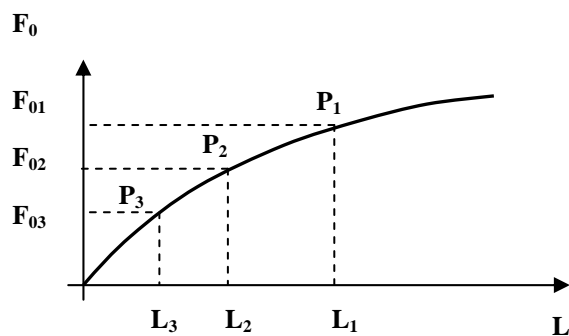


Fig. 5. Nonlinear diagram of the process

Consider three plant operating points  $P_1, P_2, P_3$  on the nonlinear diagram  $F = f(L)$  as in Fig. 5. The level values  $L_1, L_2, L_3$  can be considered the set – points of the nominal level control system.

We have identified three different models of the nonlinear process in these operating points.  $M_1$  for the high level,  $M_2$  for the medium level and  $M_3$  for the low level, where:

$$M_1 = \frac{0.08816q^{-1}}{1 - 0.94233q^{-1}}$$

$$M_2 = \frac{0.08092q^{-1}}{1 - 0.92641q^{-1}}$$

$$M_3 = \frac{0.07903q^{-1}}{1 - 0.91757q^{-1}}$$

In this case we have computed three correspondent R-S-T algorithms using a pole placement procedure. The same nominal performances are given for all systems, by a second order standard dynamic system described by  $\omega_0 = 0.05$ ,  $\xi = 0.85$  (tracking performances) and  $\omega_0 = 0.085$  and  $\xi = 0.75$  (disturbance rejection performances) respectively, with a sampling time  $T_e = 5s$ .

$$R_1(q^{-1}) = 61.824 - 46.906 q^{-1}$$

$$S_1(q^{-1}) = 1.0 - 1.0 q^{-1}$$

$$T_1(q^{-1}) = 113.378 - 158.394 q^{-1} + 59.933 q^{-2}$$

$$R_2(q^{-1}) = 65.435 - 49.171 q^{-1}$$

$$S_2(q^{-1}) = 1.0 - 1.0 q^{-1}$$

$$T_2(q^{-1}) = 123.609 - 172.686 q^{-1} + 65.341 q^{-2}$$

$$R_3(q^{-1}) = 65.592 - 49.235 q^{-1}$$

$$S_3(q^{-1}) = 1.0 - 1.0 q^{-1}$$

$$T_3(q^{-1}) = 126.582 - 176.840 q^{-1} + 66.912 q^{-2}$$

Let us consider  $P_0$  the current operating point, between  $P_1$  and  $P_2$ , near  $P_2$ . The set-point of level control system is  $L_0$  placed between  $L_1$  and  $L_2$ . According to this situation, the multi-model scheme will choose the best model  $M_2$  and will select  $C_2 (R_2, S_2, T_2)$  control algorithm. The use of this algorithm ( $R_2, S_2, T_2$ ) on the plant will assure the performances presented in the Fig. 6.

The adaptive multi-model control procedure has improved the quality of the control. The new performances obtained with the real time adaptive control system are presented in Fig. 7. In fact, the improved performances are assured using the best starting selected system ( $M_2, C_2$ ) and the adaptive procedure from  $P_2$ , towards  $P_0$ .

Each of the Fig. 6 and 7 shows at the top the evolution of the set point  $r$  and the output  $y$ , and at the bottom the evolution the control  $u$ , respectively.



Fig. 6. Performances for  $P_0$  operating point with  $(R_2, S_2, T_2)$  algorithm

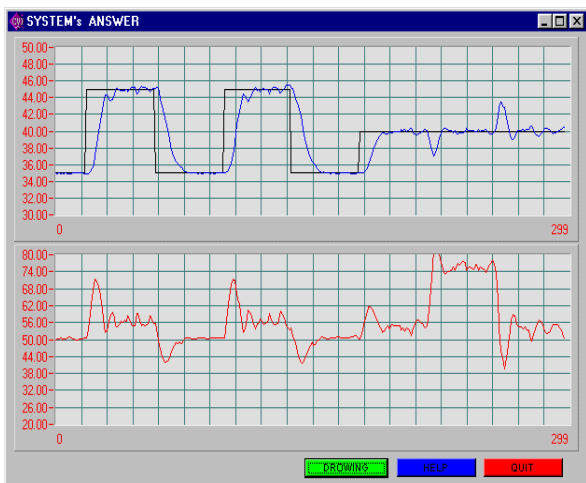


Fig. 7. Improved performances for  $P_0$  operating point using adaptive control procedure

Further, we show the close loop imposed poles placements for each  $(M_i, C_i)$  pair. The poles are imposed based on the required performances:

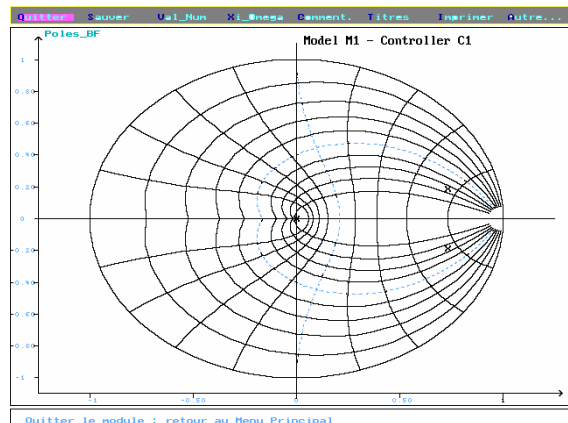


Fig 8. Close loop imposed poles for the  $(M_1, C_1)$  pair

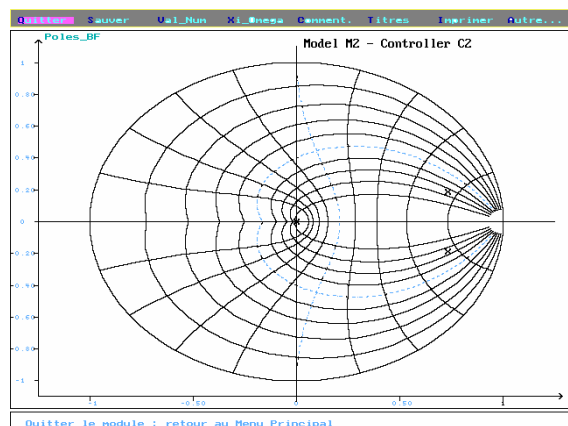


Fig 9. Close loop imposed poles for the  $(M_2, C_2)$  pair

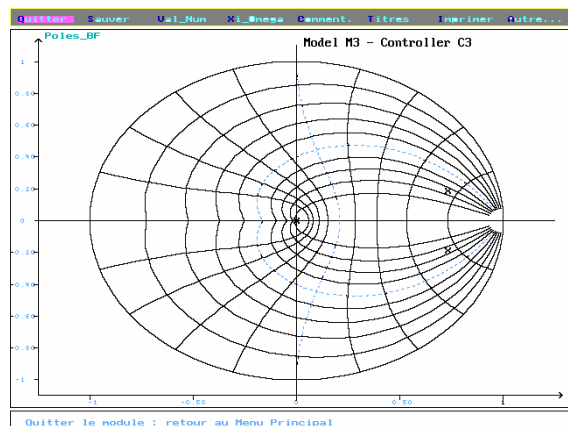
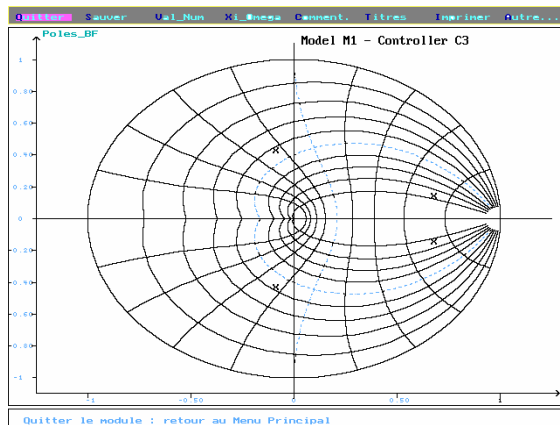
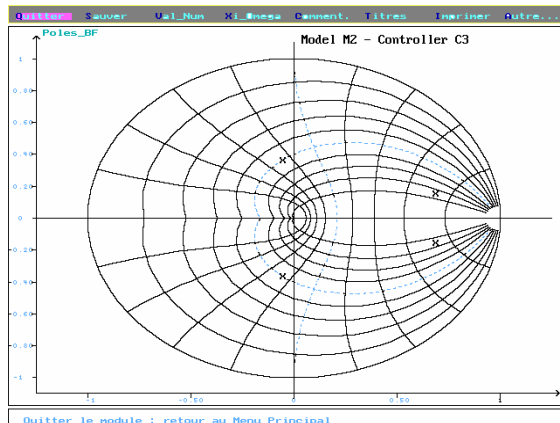
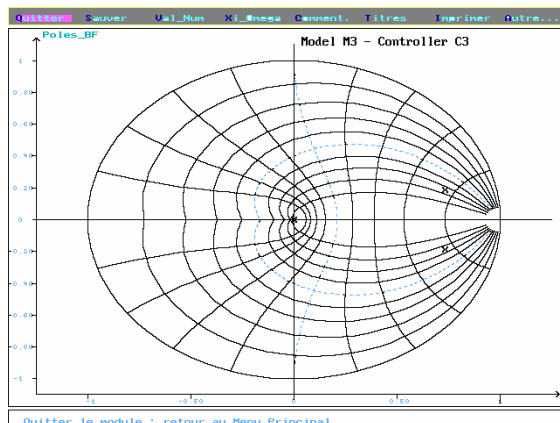


Fig 10. Close loop imposed poles for the  $(M_3, C_3)$  pair

If we use the third controller with each process, the poles “movement” will consist in:

Fig 11. Poles “movement” for the  $(M_1, C_3)$  pairFig 12. Poles “movement” for the  $(M_2, C_3)$  pairFig 13. Poles “movement” for the  $(M_3, C_3)$  pair

The poles in figure 10 will show no movement compared to the poles in figure 13.

## 6 Acknowledgements

An application of the multiple models adaptive control to a nonlinear model plant has been presented.

A mechanism based on the performance model-error criterion for the choice of the best model in switching phase is considered.

The closed loop identification algorithm (CLOE) and R-S-T adaptive control algorithm is used.

The multiple model adaptive control procedure proposed has the following advantages: a more precise model is chosen for the closed loop operating system, the R-S-T adaptive control ensures very good real time results for closed loop nonlinear system.

It can be appreciate that the multiple models adaptive control can be recommended to improve the performances of the nonlinear control systems.

## 7 References

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