STRUCTURAL PROPERTIES OF SWITCHING BOND GRAPH

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Abstract

Switching systems are very common in various engineering fields (e.g. hydraulic systems with valves,.., electric systems with diodes, relays,…, mechanical systems with clutches...). Such systems are a particular case of hybrid systems. These systems are characterized by a Finite State Automaton (FSA) and a set of dynamic systems, each one corresponding to a state of the FSA. The change of states can be either controlled or autonomous. The aim of this work is to investigate the structural controllability for controlled switching linear systems modelled by bond graph.

Several concepts appeared in the last decade addressing the controllability problem of these systems: controllable sublanguage concept [9], hybrid controllability concept [10], between-block controllability concept [11]. Controlled switching linear systems (CSLS) on which we focus in this work belong to the hybrid controllability concept as they address a reachability problem of hybrid states.

In the other hand, the bond graph concept is an alternate representation of physical systems. Some recent works permit to highlight structural properties**.** In [7]**,** the structural controllability property is studied using simple causal manipulations on the bond graph model. The objective of this work is to extend these properties to CSLS systems. The bond graph structure junction contains informations on the type of the elements constituting the system, and how they are interconnected, whatever the numerical values of parameters.

The structural controllability of CSLS is studied using simple causal manipulations on the bond graph model. For that, formal representation of structural controllability subspace, is given for bond graph model. It is calculated using causal manipulations. The base of this subspace is used to propose a graphical procedure to study the structural controllability.

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Keywords: Hybrid systems, Switching systems, Bond graph, Structural controllability.

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1 Introduction

A broad class of hybrid systems is composed of physical processes with switching devices. Such processes are called switching systems and are very common in various engineering fields (e.g. hydraulic systems with valves,.., electric systems with diodes, relays,…, mechanical systems with clutches...). These systems are characterized by a Finite State Automaton (FSA) and a set of dynamic systems, each one corresponding to a state of the FSA. The change of states can be either controlled or autonomous. Various researchers investigated this problem using the bond graph tool [1,2,3,4,5,6]. The ideal and the non-ideal approaches are used :

- In the non-ideal approach, switches are modelled as resistive elements associated with modulated transformer. The modulation is done using a boolean variable.

- In the ideal approach, switches commutate instantaneously. Each switch is modelled as a null source: effort source for a closed switch state, and flow source for an open one. This approach is used in this work.

Several concepts appeared in the last decade addressing the controllability problem: controllable sublanguage concept [9], hybrid controllability concept [10], between-block controllability concept [11]. Controlled switching linear systems (CSLS) on which we focus in this work belong to the hybrid controllability concept as they address a reachability problem of hybrid states.

The aim of this work is to investigate the structural controllability for controlled switching linear systems modelled by bond graph. This paper is organized as follows: The second section, formulates the CSLS controllability. Section three recalls some background about bond graph modelling of hybrid systems with ideal switches. In section four the structural controllability of these systems is discussed using bond graph approach and using algebraic characterization. Graphical conditions and procedures are proposed. Finally, a simple example illustrates the previous results is proposed.

2 Controllability of controlled switching linear systems

Consider a Controlled Switching Linear Systems [8], given by equation (1):

$$\dot{x}(t) = A(\sigma(t))x + B(\sigma(t))u \tag{1}$$

Where $x \in R^n$ is the state variable, $u \in R^m$ is the input variable, $\sigma : R \to Q = \{\sigma_i, i \in \{1,\cdots,q\}\}$ is a piecewise constant switching function and (σ_i, x) the hybrid state. According to values of $\sigma(t)$, there exists q configurations, $\sigma_i \in \{\sigma_1, \cdots \sigma_q\}$. So, $A(\sigma_i) \in R^{n\times n}$ and $B(\sigma_i) \in R^{n\times m}$.

The characteristics of CSLS are:

▪ The dynamical subsystem within each mode has a linear time invariant form,

▪ The admissible region of operation within each mode is the whole state and input space,

Assumptions

1) We suppose that $A(\sigma_i)$ and $B(\sigma_i)$ matrices are constant on $[t_0, t_0 + \tau)$, where $\tau \ge \tau_{\min} > 0$, and constant $\tau_{\min}$ is an arbitrarily small and independent of mode i. For instance, suppose that the dynamics in (1) are given by $\dot{x} = A(\sigma_i)x + B(\sigma_i)u$ over the finite time interval $[t_k, t_{k+1})$. At time t_{k+1} the dynamic in interval $[t_{k+1}, t_{k+2})$ is given by $\dot{x} = A(\sigma_j)x + B(\sigma_j)u$.

2) We assume that the state vector $x(t)$ does not jump discontinuously at t_{k+1}.

Under these assumptions, the CSLS controllability of (1) was defined:

Definition 1 [8] Given any pair of hybrid states, (σ_0, x_0) and (σ_q, x_q), if there exists a timed mode-switching set $\{(\sigma_{i-1}, t_i, \sigma_i)\}_{i=1}^{q}$ and a corresponding piecewise continuous-finite input signal $u(t)$, such that system (1) evolving under these two distinct inputs is reachable from (σ_0, x_0) to (σ_q, x_q) within a finite time interval, then the considered system (1) is controllable, otherwise, system (1) is uncontrollable.

2.1 Necessary and sufficient algebraic condition

Firstly, we assume that one switching-mode set is known as $\{\sigma_{i_l}\}_{l=1}^{k}$, where $i_l \ne i_{l+1}$ for $l = 1,\cdots,k-1$.

Let us define the (n, mn^k) matrix $Œ^k(i_1,\cdots,i_k) \,\hat{=}\, [A_{i_k}^{j_k} \cdots A_{i_2}^{j_2} A_{i_1}^{j_1} B_{i_1}]_{j_k,\cdots,j_1 \in \{0,\cdots,n-1\}}$.

Based on the definition of Œ we construct a new matrix Æ as follows:

$$Æ^0(i) \,\hat{=}\, Œ^1(i) = W_i, \ldots,$$
$$Æ^k(i) \,\hat{=}\, [Œ^{k+1}(i, i_1, \cdots, i_k)]_{i_1,\cdots,i_k \in \{1,\cdots,q\}}$$

With $i_1 \ne i, \cdots, i_k \ne i_{k-1}$.

The joint controllability matrices can be defined as:

$$\bar{W}^0 = [Æ^0(1) \cdots Æ^0(q)], \ldots, \bar{W}^k = [Æ^k(1) \cdots Æ^k(q)] \tag{2}$$

$\bar{W}^k$ is the k^{th}-order joint controllability matrix of the system (1). There exists a joint controllability coefficient k_r of the system, defined in [8]:

$$k_r \,\hat{=}\, \arg\min_{l}(rank(\bar{W}^l) = rank(\bar{W}^{l+1}))$$

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Theorem 1 [12] System (1) is controllable, if and only if $rank(\bar{W}^{k_r}) = n$.

This theorem can be interpreted using geometric approach. Let us firstly recall some concepts.

Definition 2 (Invariant subspace) [15] Given a matrix A and a subspace $\mathbf{B} = Im(B)$, the invariant subspace $\langle A|\mathbf{B}\rangle$ is defined by:

$$\langle A|\mathbf{B}\rangle \overset{def}{=} \sum_{i=1}^{n} A^{i-1}\mathbf{B} = \mathbf{B} + A\mathbf{B} + \cdots + A^{n-1}\mathbf{B} \tag{3}$$

For system (1), [15] defined a subspace sequence as follows:

$$\upsilon_1 = \sum_{i=1}^{q} \langle A_i | \mathbf{B}_i \rangle,\ \upsilon_{j+1} = \sum_{i=1}^{q} \langle A_i | \upsilon_j \rangle \quad j = 1, 2, \cdots$$

$$\text{and} \qquad \upsilon = \sum_{k=1}^{\infty} \upsilon_k \tag{4}$$

The following proposition shows the relationship between the previously defined subspaces and the joint controllability matrix.

Proposition 1 [13] The subspace υ (equation 4) and the k^{th}-order joint controllability matrix $\bar{W}^k$ are linked by the following relation: $\mathrm{Im}\left(\bar{W}^k\right) = \upsilon$.

Based on this proposition, a geometric necessary and sufficient condition is introduced.

Proposition 2 System (1) is controllable, if and only if $\upsilon = R^n$.

Proof. Easy by using theorem 1 and proposition 1.

2.2 Controllability subspace basis

In this subsection, we give a procedure proven in [13] to calculate υ.

Denote the nested subspaces as $\mathbf{W}_0 = \mathbf{B}_1 + \cdots + \mathbf{B}_q$,

$\mathbf{W}_j = \mathbf{W}_{j-1} + \sum_{k=1}^{q} A_k \mathbf{W}_{j-1}\ j = 1, 2, \ldots,$ and

$\mathbf{W} = \sum_{j=0}^{\infty} \mathbf{W}_j$. We have $\mathbf{W}_0 \subset \mathbf{W}_1 \subset \mathbf{W}_2 \subset \cdots \subset \mathbf{W}$ and $\upsilon = \mathbf{W}$. Note that if $\mathbf{W}_j = \mathbf{W}_{j+1}$ for some j, then $\mathbf{W}_k = \mathbf{W}_j$ for $k \geq j$ and further $\mathbf{W}_j = \mathbf{W} = \upsilon$. This fact together with $\dim \mathbf{W} \leq n$ imply that $\mathbf{W}_{n-n_0} = \mathbf{W} = \upsilon$,

where $n_0 = \dim \mathbf{W}_0$.

Denote $\rho = \min\{k : \mathbf{W}_k = \upsilon\} \leq n - n_0$ and $n_k = \dim \mathbf{W}_k \quad k = 1, \ldots, \rho$.

A basis of υ can be constructed according to the following procedure:

Procedure 1

1) Choose a group of base vectors $\eta_1, \ldots, \eta_{s_1}$ in $\mathbf{B}_1$,

2) Expand them to $\eta_1, \ldots, \eta_{s_1}, \eta_{s_1+1}, \ldots, \eta_{s_2}$ which form a basis of $\mathbf{B}_1 + \mathbf{B}_2$,

3) Repeat this operation, and write a basis $\eta_1, \ldots, \eta_{n_0}$ of $\mathbf{W}_0$,

Because
$\mathbf{W}_1 = \mathbf{W}_0 + \mathrm{Im}\{A_j\eta_k,\ j = 1, \ldots, q,\ k = 1, \ldots, n_0\}$
$= \mathrm{Im}\{\eta_1, \ldots, \eta_{n_0},\ A_j\eta_k,\ j = 1, \ldots, q,\ k = 1, \ldots, n_0\}$

4) Write a basis $\eta_1, \ldots, \eta_{n_1}$ of $\mathbf{W}_1$ by searching the set $\{\eta_1, \ldots, \eta_{n_0},\ A_j\eta_k,\ j = 1, \ldots, q,\ k = 1, \ldots, n_0\}$ from left to right,

5) Repeat the operation, and write a basis $\eta_1, \ldots, \eta_{n_0}, \ldots, \eta_{n_{l-1}+1}, \ldots, \eta_{n_l}\}$ for $\mathbf{W}_l$.

Because
$\mathbf{W}_{l+1} = \mathbf{W}_l + \mathrm{Im}\{A_j\eta_k,\ j = 1, \ldots, q,\ k = n_{l-1}+1, \ldots, n_l\}$
$= \mathrm{Im}\{\eta_1, ..., \eta_{n_l}, A_j\eta_k,\ j = 1, ..., q,\ k = n_{l-1}+1, ..., n\}$

6) By searching the set $\{\eta_1, \ldots, \eta_{n_l}, A_j\eta_k,\ j = 1, \ldots, q,\ k = n_{l-1}+1, \ldots, n_l\}$ and write a basis $\eta_1, \ldots, \eta_{n_0}, \ldots, \eta_{n_{l-1}+1}, \ldots, \eta_{n_l}, \eta_{n_l+1}, \ldots, \eta_{n_{l+1}}$ for $\mathbf{W}_{l+1}$.

7) Write $\upsilon = \mathrm{Im}\{\eta_1, \ldots, \eta_{n_0}, \ldots, \eta_{n_{\rho-1}+1}, \ldots, \eta_{n_\rho}\}$ (5)

Remark. From the above analysis; a basis for υ is of the form

$$\begin{aligned} \{ & b_1, A_{i_{1,1}}b_1, A_{i_{2,1}}A_{i_{1,1}}b_1, \cdots, A_{i_{r_1,1}}\cdots A_{i_{1,1}}b_1, \cdots, \\ & b_2, A_{i_{1,2}}b_2, A_{i_{2,2}}A_{i_{1,2}}b_1, \cdots, A_{i_{r_2,2}}\cdots A_{i_{1,2}}b_2, \cdots, \\ & b_{n_0}, A_{i_{1,n_0}}b_{n_0}, A_{i_{2,n_0}}A_{i_{1,n_0}}b_{n_0}, \cdots, A_{i_{r_{n_0},n_0}}\cdots A_{i_{1,n_0}}b_{n_0} \} \end{aligned} \tag{6}$$

Where $b_k \in \mathbf{W}_0, r_k \geq 0, 1 \leq i_{l,k} \leq q, l = 1, \cdots, r_{n_0}$, $k = 1, \cdots, n_0$. Because the number of vectors in (6) is not more than n; there are at most n different subsystems whose parameters appear in (6). That is to say; for controllability issues; we may assume $q \leq n$ without loss of generality.

3 Bond graph approach

The bond graph structure junction contains informations on the type of the elements constituting the system, and how they are interconnected, whatever the numerical values of parameters. The structure junction of a switching bond graph can be represented by figure 1. Five fields model the components behaviour, 4 that belong to the standard bond graph formalism; - source field which produces energy, - R field which dissipates it, - I and C field which can store it, and the Sw field that is added for switching components.

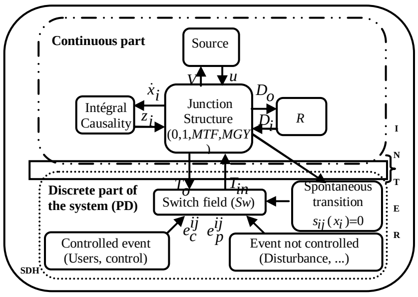

Fig 1. Structure junction

Assumptions:

1) To take into account the absence of discontinuities, we suppose that is no elements in derivative causality in the bond graph model in integral causality, before and after switching. It can be obtained by assuming that switches commutated by pairs.

2) A switch is considered as a discrete control;

Using the ideal approach, a switch can be modelled as shown in figure 2:

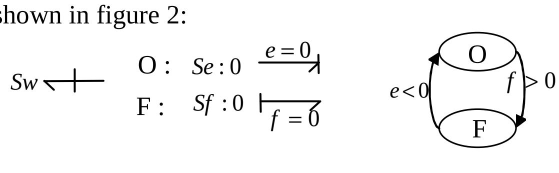

Fig 2. Representation of ideal switch

The corresponding junction matrix is given by equation 7 [2]:

$$\begin{pmatrix} \dot{x} \\ D_0 \\ T_{0_i} \\ y \end{pmatrix} = \begin{pmatrix} S_{11} & S_{13} & S_{14} & S_{15} \\ -S_{13}^T & S_{33} & S_{34} & S_{35} \\ -S_{14}^T & -S_{34}^T & S_{44} & S_{45} \\ S_{51} & S_{53} & S_{54} & S_{55} \end{pmatrix} \begin{pmatrix} z \\ D_i \\ T_{in_i} \\ u \end{pmatrix} \tag{7}$$

$D_i = LD_0$, L is a positive matrix. Let assume that $H = L(I - S_{33}L)^{-1}$ is an invertible positive matrix. Then the second row leads to $D_i = -HS_{13}^T Fx + HS_{34}T_{in_i} + HS_{35}u$

The third line of (7) gives:

$$T_{0_i} = (-S_{14}^T + S_{34}^T HS_{13}^T)Fx + (S_{44} - S_{34}^T HS_{34})T_{in_i} + (S_{45} - S_{34}^T HS_{35})u$$

The substitution in the first line of (7) gives: $\dot{x} = (S_{11} - S_{13}HS_{13}^T)Fx + (S_{14} - S_{13}HS_{34})T_{in_i} + (S_{15} + S_{13}HS_{35})u$

Then, we have: $\dot{x} = A_i x + B_{ci}u + B_{di}T_{in_i}$ (8)

Where $A_i = (S_{11} - S_{13}H\,S_{13}^T)F$, $B_{ci} = S_{15} + S_{13}\,H\,S_{35}$ and $B_{di} = S_{14} + S_{13}\,H\,S_{34}$.

After the commutation, the new inputs and outputs of the junction structure associated with switches become $T_{in}(\sigma)$ and $T_0(\sigma)$, which can be related with T_{in_i} and T_{o_i}. $\Lambda(\sigma)$ is a square diagonal matrix whose diagonal elements are the components of σ in the new mode.

$$\begin{cases} T_{in}(\sigma) = (I - \Lambda(\sigma))T_{in_i} + \Lambda(\sigma)T_{o_i} \\ T_0(\sigma) = \Lambda(\sigma)T_{in_i} + (I - \Lambda(\sigma))T_{o_i} \end{cases} \tag{9}$$

Using (9) and (7) we have:

$$\begin{cases} \dot{x} = A(\sigma)x + B_c(\sigma)u + B_d(\sigma)T_{in}(\sigma) \\ T_0(\sigma) = C_d(\sigma)x + D_c(\sigma)u + D_d(\sigma)T_{in}(\sigma) \\ D_o = C_1(\sigma)x + D'_c(\sigma)u + D'_d(\sigma)T_{in}(\sigma) \end{cases} \tag{10}$$

$A(\sigma) = [(S_{11} - S_{13}H\,S_{13}^T) + (S_{14} + S_{13}H\,S_{34})k_2k_5(S_{14}^T - S_{34}^T H\,S_{13}^T)]F$

$B_d(\sigma) = [(S_{14} + S_{13}HS_{34})k_1 + (S_{14} + S_{13}HS_{34})k_2k_5(S_{34}^T HS_{34}k_1 - S_{44}k_1 + k_3)]$

$B_c(\sigma) = [(S_{15} + S_{13}H\,S_{35}) + (S_{14} + S_{13}H\,S_{34})k_2k_5(S_{34}^T HS_{35} - S_{45})]$

$C_d(\sigma) = [k_5(S_{14}^T - S_{34}^T HS_{13}^T)]$, $D_c(\sigma) = k_5(S_{34}^T HS_{35} - S_{45})$,

$D_d(\sigma) = [k_5(S_{34}^T HS_{34}k_1 - S_{44}k_1 + k_3)]$,

$C_1(\sigma) = L^{-1}H[-S_{13}^T + S_{34}k_2k_5(S_{14}^T - S_{34}^T H\,S_{13}^T)F]$,

$D'_d(\sigma) = L^{-1}H[S_{34}k_1 + S_{34}k_2k_3(S_{34}^T HS_{34}k_1 - S_{44}k_1 + k_3)]$,

$D'_c(\sigma) = L^{-1}H[S_{35} + S_{34}k_2k_3(S_{34}^T HS_{34} - S_{45})]$,

$k_1(\sigma) = [I + (I - 2\Lambda(\sigma))^{-1}\Lambda(\sigma)]$,

$k_3(\sigma) = -(I - 2\Lambda(\sigma))^{-1}\Lambda(\sigma)$,

$k_2(\sigma) = [I - (I - 2\Lambda(\sigma))^{-1}(I - \Lambda(\sigma)]$,

$k_4(\sigma) = (I - 2\Lambda(\sigma))^{-1}(I - \Lambda(\sigma))$, and

$k_5(\sigma) = [(-S_{34}^T HS_{34}k_2(\sigma) + S_{44}k_2(\sigma) - k_4(\sigma)]^{-1}$.

Therefore, for N switches, we have q modes, and

$$\begin{cases} \dot{x} = A_1x + B_{c1}u + B_{d1}T_{in_1} & t \in [t_0, t_1) \\ \quad\vdots & \quad\vdots \\ \dot{x} = A_qx + B_{cq}u + B_{dq}T_{in_q} & t \in [t_{q-1}, t_q) \end{cases} \tag{11}$$

4 Structural controllability

The bond graph concept is an alternate representation of physical systems. Some recent works permit to highlight structural properties of these systems [7,5]. In [7], the structural controllability property is studied using simple causal manipulations on the bond graph model. It is shown that the structural rank concept is somewhat different for bond graph models because it is more precise than for other representations. Our objective is to extend these properties to CSLS systems.

In the following we note that:

-BG: acausal (without causality) bond graph model

-BGI: bond graph model when the preferential integral causality is affected

-BGD: bond graph model when the preferential derivative causality is affected

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- t^i : the number of elements in integral causality in BGD_i. *i* indicate the mode *i*.

- t_s^i : the number of elements in integral causality in BGD_i, when a dualization of the maximum number of continuous input sources is applied (in order to eliminate elements in integral causalities).

- $t_{Sw_s}^{ij}$: the number of elements remaining in integral causality in BGD_i, when a dualization of the maximum number of continuous input sources is applied (in order to eliminate elements in integral causalities) and a dualization of the maximum number of discrete input sources is applied (in order to eliminate these integral causalities).

Let us recall the structural controllability of LTI systems (case *q*=1).

Theorem 2 [7] The system $\sum_i (A_i \; B_i)$ is structurally state controllable if and only if:

- On the BGIi, all dynamical elements in integral causality are causally connected with a continuous control.

- BG-$rank[A_i \; B_i] = n$.

Property 1 [7]

BG-$rank[A_i \; B_i]$=$rank(S_{11} \, S_{13} \, S_{15}) = n - t_s^i$.

In the next step structural controllability of CSLS modelled by bond graph is studied. For that, formal representation of structural controllability subspace, denoted as R_0, is given for BG model. It is calculated using causal manipulations. The base of this subspace is used to propose a procedure to study the structural controllability.

4.1 Graphical necessary and sufficient condition

On the BGD_i (and dualization of inputs sources) there exists t_s^i elements remaining in integral causality and $(n - t_s^i)$ elements in derivative causality.

t_s^i algebraic equations can be written (equation 12):

$$g_k^i - \sum_r \alpha_r^{ik} g_r^i = 0 \tag{12}$$

- g_k^i is either an effort variable e_r for I-element in integral causality or a flow variable f_r for C-element in integral causality,

- g_r^i is either an effort variable e_r for I-element in derivative causality or a flow variable f_r for C-element in derivative causality,

- α_r^{ik} is the gain of the causal path between the k^{th} I or C-elements in integral causality and the r^{th} I or C-elements in derivative causality.

Let us consider the t_s^i row vectors z_k^i $(k = 1, \cdots, t_s^i)$ whose components are the coefficients of the variables g_k^i and g_r^i in equation (12).

Property 2 [6] The t_s^i row vectors z_k^i $(k = 1, \cdots, t_s^i)$ are orthogonal to the structural controllability subspace vectors of the i^{th} mode. We write $Z_i = (z_k^i)_{k=1,\cdots,t_s^i}$ and $R_0^{i\perp} = \mathrm{Im}(Z_i)$.

$R_0^{i\perp}$: uncontrollable subspace in mode i, used to check orthogonality.

Procedure 2: Calculation of $R_0^{i\perp}$

1) On the BGD_i, dualize the maximum number of input sources in order to eliminate the elements remaining in integral causality,

2) For each element in integral causality, write the algebraic relation with elements in derivative causality (equation 12),

3) Write a row vector z_k^i for each algebraic relation with the causal path gains. (equation 12),

In order to calculate a R_0^i basis, it is enough to find $(n - t_s^i)$ independent column vectors $w^{ir} (r = 1, \cdots, n - t_s^i)$ **.** These vectors are gathered in the matrix $W^i = (w^{ir})_{r=1,\cdots,n-t_s^i}$.

In the same manner, from the BGD_i (and dualization of inputs sources) $(n - t_s^i)$ algebraic relations can be calculated (13).

$$g_r^i - \sum_k \gamma_k^{ir} g_k^i = 0 \tag{13}$$

- g_r^i is either an flow variable f_r for I-element in derivative causality or a effort variable e_r for C-element in derivative causality,

- g_k^i is either an flow variable f_r for I-element in integral causality or a effort variable e_r for C-element in integral causality,

- γ_k^{ir} is the gain of the causal path between the r^{th} element in derivative causality and the k^{th} element in integral causality.

Suppose now the $(n - t_s^i)$ column vectors w^{ir} whose components are the coefficients of the variables g_r^i and g_k^i in equation (13)**.**

Procedure 3 : Calculation of R_0^i

1) On the BGD_i, dualize the maximum number of

continuous control in order to eliminate the elements in integral causality.

2) For each element remaining in derivative causality, write the algebraic relation with elements in integral causality, (equation 13),

3) Write a column vector w^{ir} for each algebraic relation with the causal path gains, (equation 13),

with $R_0^i = \mathrm{Im}(W^i)$.

From the BGD$_i$ (and dualization of inputs sources), the following relation can be calculated for each switch:

$$T_{o_i} - g'^i_r - \sum_k \gamma_k^{jr} g'^i_k = 0 \qquad (14)$$

- T_{o_i} is the variable on the switch, outgoing of the junction structure,

- g'^i_r and g'^i_k can be effort or flow,

- γ_k^{jr} is defined in equation 13.

From (14) we propose the invariants for the BGD:

Proposition 3 For the hybrid system (1), the invariant associated to each switch for BGD$_i$ is given by the inequality constraints relating to the i^{th} mode:

$$Inv^d(\sigma_i): \; T_{o_i} = g'^i_r + \sum_k \gamma_k^{jr} g'^i_k > 0 \qquad (15)$$

At instant of commutation, from equation 14 and after the annulation of T_{o_i}, N conditions can be given:

$$g'^i_r + \sum_k \gamma_k^{jr} g'^i_k = 0 \qquad (16)$$

g'^i_r, g'^i_k and γ_k^{jr} are defined in equation 13.

Suppose now the $t_{Sw_s}^{(i-1)\to i}$ column vectors $w_{Sw_{js}}^{(i-1)\to i}$ ($j = 1, \dots, N$) whose components are the coefficients of the variables g'^i_r and g'^i_k in equation (16).

Procedure 4: Calculation of R_0

1) After dualization of the maximum number of input sources in BGD$_i$, write the relation between each switch element and the dynamical elements,

2) Deduce the $t_{Sw_s}^{(i-1)\to i}$ invariants for the corresponding BGD$_i$,

3) Write the conditions of commutation using equation 16,

4) Write a column vector $w_{Sw_{js}}^{(i-1)\to i}$ ($j = 1, \cdots, N$) for each algebraic relation with the causal path gains,

5) Check if $z_k^i . w_{Sw_{js}}^{(i-1)\to i} = 0$, and write $R_0 = \mathrm{Im}(w^{1k_1} \; w^{1\to 2} \cdots w^{q-1\to q})$, With

$$w^{i-1\to i} = [w^{ik_i} \; w_{Sw_{js}}^{(i-1)\to i}]_{k_i = 1,\cdots,n-t_s^i,\; i=1,\cdots,q}.$$

w^{1k_1}: The basis of controllability subspace of initial mode.

Remark. If the sequence of commutation is not ordered**,** then

$$w^{i-1\to i} = [w^{ik_i} \; w_{Sw_{js}}^{(i-1)\to i} \cdots w_{Sw_{js}}^{(i-r)\to i}]_{k_i = 1,\cdots,n-t_s^i,\; i,r=1,\cdots,q \;\; and\; i\neq r}$$

Proposition 4 System (1) is structurally controllable, if and only if $rank(w^{1k_1} \;\; w^{1\to 2} \cdots w^{q-1\to q}) = n$.

5 Example

Let us consider the following acausal BG model (figure 3):

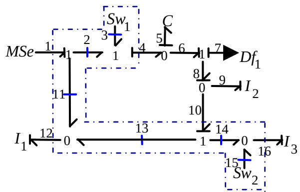

Fig 3: The acausal BG

We have two complementary switches, then we have two possible configurations: mode 1 (Sw_1 :closed, Sw_2 open) and mode 2 (Sw_1 :open, Sw_2 :closed).

The bond graph models in integral causality of mode 1 and 2 are shown in figure 4:

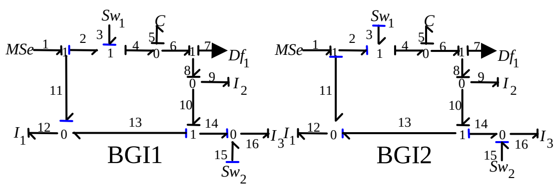

Fig 4: The BGI1 and BGI2

There are four state variables, P_{I_i} on I_i ($i = 1, \cdots, 3$), q_c on C. Figure 5 presents the bond graphs in derivative causality after the dualization of inputs sources :

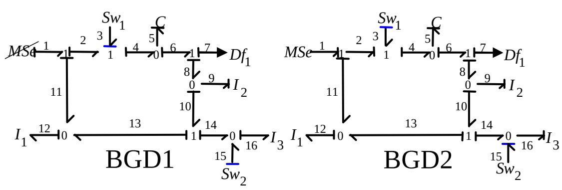

Fig 5: a) BGD1+dualization of sources (mode 1), b) BGD2+dualization of sources (mode 2)

■ Calculation of W^i (application of procedure 3)

▪ Calculation of W^1 (mode 1)

The element I_2 is in integral causality, we can write $e_{I_1} - e_{I_2} + e_{I_3} = 0$, thus $z_1^1 = (1 -1 1\,0)$.

The algebraic equations corresponding to I_1 and I_3 are given by: $f_{I_1} + f_{I_2} = 0$, $f_{I_3} + f_{I_2} = 0$. Then $w^{11} = (1 1 0\,0)^t$, $w^{12} = (0 1 1\,0)^t$. The dynamical element C is not causally connected with I_2, we can write $e_c = 0$. The corresponding vector is $w^{13} = (0\,0\,0\,1)^t$. and $R_0^1 = \mathrm{Im}\{w^{11}, w^{12}, w^{13}\}$.

▪ Calculation of W^2 (mode 2)

The element I_3 is in integral causality and not causally connected with I_2, we can write $e_{I_3} = 0$, thus $z_1^2 = (0\,0\,1\,0)$. The element I_2 is in integral causality, we have $e_{I_1} - e_{I_2} = 0$, thus $z_2^2 = (1 -1\,0\,0)$.

The algebraic equation corresponding to the element I_1 is given by: $f_{I_1} + f_{I_2} = 0$, then $w^{21} = (1 1 0\,0)^t$. The dynamical element C is not causally connected with I_2 and I_3, we can write $e_c = 0$, The corresponding vector is $w^{22} = (0\,0\,0\,1)^t$ and $R_0^2 = \mathrm{Im}\{w^{21}, w^{22}\}$.

■ Inequality constraints (application of proposition 3)

The invariants ($Inv^d(\sigma_1)$ and $Inv^d(\sigma_2)$) can be computed according to equation (15):

Mode 1: $f_{Sw_1} = f_{I_1} + f_{I_2} > 0$, $e_{Sw_2} = e_{I_3} > 0$, mode 2: $e_{Sw_1} = e_{I_1} > 0$, $f_{Sw_2} = f_{I_3} - f_{I_1} > 0$

■ Calculation of $w^{i-1 \to i}$ (Application of procedure 4, steps 1, 2, 3, 4)

We suppose that mode 1 is the initial mode, therefore it is characterized by its controllable subspace $R_0^1 = \mathrm{Im}(w^{11}, w^{12}, w^{13})$ and its inequality constraints $f_{Sw_1} = f_{I_1} + f_{I_2} > 0, e_{Sw_2} = e_{I_3} > 0$.

After commutation, we have: $-z_1^2 = (0\,0\,1\,0)$, $z_2^2 = (1 -1\,0\,0)$, $w^{21} = (1 1 0\,0)^t$, $w^{22} = (0\,0\,0\,1)^t$ and

$w_{Sw_{1s}}^{1\to 2} = (1 1 0\,0)^t$, because $z^2.w_{Sw_{2s}}^{1\to 2} = 0$, thus

$$R_0^{1\to 2} = \mathrm{Im}(\underbrace{w^{21}, w^{22}, w_{Sw_{1s}}^{1\to 2}}_{w^{1\to 2}}) = \mathrm{Im}\left\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\\0\end{bmatrix}\right\}$$ and

$rank(w^{1\to 2}) = 3$.

[2→1]:

$$R_0^{2\to 1} = \mathrm{Im}(\underbrace{w^{11}, w^{12}, w^{13}, w_{Sw_{2s}}^{2\to 1}}_{w^{2\to 1}}) = \mathrm{Im}\left\{\begin{bmatrix}1\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \begin{bmatrix}-1\\0\\1\\0\end{bmatrix}\right\}$$

■ Calculation of W (Application of procedure 4, step 5)

$R_0 = \mathrm{Im}(w^{1k_1} \quad w^{1\to 2} w^{2\to 1}) = R^4$, so the system is structurally controllable.

6 Conclusion

The structurally controllability of CSLS systems was presented using simple causal manipulations on the BG. Thus, formal calculation enables us to know the reachable variables; its checking is immediate on the BGI. On the other hand the BGD enables us to characterize graphically the structural controllability subspaces relating to each mode. A necessary and sufficient condition was given by exploiting these various bases.

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