IMAGE PROCESSING FOR BIOLOGIC MODELS USING FRACTAL TECHNIQUES

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Abstract

The article discusses problems of classification based on the fractal theory with applications in biology. Here are introduced the necessary notions for defining and quantitative evaluation of fractals and an algorithm for fractal dimension computation based on biofractal contours processing is also presented. The actual application is dedicated to the analyse the particularities of some species from Gentianaceae family, with the purpose of establishing their affiliation to the Gentina genus, knowing the fact that, up to the present, there had been used only evaluations based on the distinctive morphological characteristics. Concretely, there were extracted window-images from the rind and the central cylinder of the root and stem and also from the mesophyll and leaf nervure/rib. The contours of the window-images were processed with the "box-counting" algorithm in order to establish the fractal dimensions for the analysed sections. The algorithm based on the „box-counting" method offers two major advantages: it is easy to implement in case of using a computer and can be applied for images no matter how complex. We consider that, by acquiring samples from more sections of a species (the studied one) for the statistical processing of the data, will lead to very precise characterizations of that species, for the first time in botanic. The results are encouraging for the development of fractal techniques not only in botanic, but in other biology domains.

Keywords: fractals, fractal dimension, biologic models.

Presenting Author's biography

Andreea Udrea is a teaching assistant at the Automatics and Computer Science Faculty, University "Politehnica" of Bucharest. Her teaching areas include Object Oriented Programming and Control Systems for Continuous Processes. She also carries out research activities and handles research projects within the ACPC Research Centre. She graduated the same Faculty in September 2006. She completed her diploma project – "Control of an autonomous electric vehicle" – at Ecole Polytech Lille, France. Now she is attending the master courses in Advanced Automatics.

1 Intoduction

In order to understand the surrounding world, the natural sciences have progressed focusing on the simplest forms of representation, in accordance with the principle: simplicity explains complexity. Sometimes the research turned away from the direct study of nature, of the details of reality and limited themselves to studies based on general, approximate and linear expressions.

In the '60s, the mathematician Mandelbrot proposes/aims to study the complex irregular forms in nature that he names fractals and builds the bases of fractal (non euclidan) geometry.

Fractals are objects with irregular, auto-similar features, with details that can be noticed at any scale of representation.

The appearance of fractal geometry marks the return of scientific knowledge to the real world.

It was rather easy to observe that the forms of rivers, mountains, the Earth in its details are of fractal type. Important examples of fractals in botanic – that we shall name fitofractals - are the leafs of a tree, the structure of the tissue from a plant's stem or root section, the forms and contours of the cells etc.

With the aid of fractal geometry, the growth and ramification models from the plants world can be explained and reproduced (fig. 1) by using strings of ordered characters and simple operations of translation and rotation.

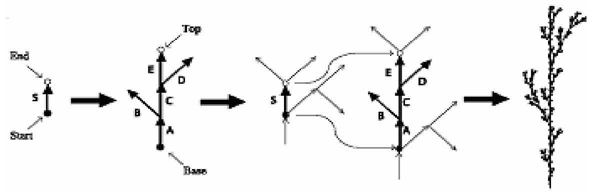


Fig.1 The construction of a L-system that can be applied to ramification types of plants [1]

An important feature of fractal objects is the dependence between their dimension and the used measure unit (fig2). By choosing a finer measure unit, an irregular contour can be better approximated, with a finer accuracy, this way better showing its details.

Richardson noticed this fact for the first time. Wanting to know the length of the borderline between Spain and Portugal, he consulted the documents from the archives of both countries. In the Spanish encyclopaedia it was written that the borderline had a length of 987km, while in the Portuguese encyclopaedia the length was approximated to 1214km. The explication of the strange phenomenon was that two different measure units were used; the smaller unit could go over more border details, and so obtain a finer measurement, like in the Portugal case.

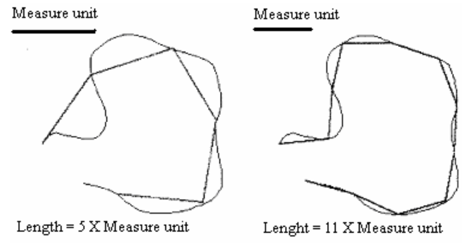


Fig. 2 The dependence of the length of a curve upon the used measure unit

The dependence upon the used scale measure makes the fractal objects difficult to measure in the classic (eucledian) geometry context.

The Euclidian dimension D is given by the number of coordinates needed to define any of the points of the object, or, more exactly, the dimension of the Euclidian space in which the analysed object can be submerged into: the line in a plane, the cone in a three dimensional space.

The topological dimension T is defined by the local properties of the analysed object and corresponds to the concept that the dimension of a point is 0, the dimension of a line, thin curve is 1, the surfaces have the dimension equal with 2, volumes with 3 and so on, without taking into account any bigger dimension of the Euclidian space in which these forms were submerged.

Alongside with the apparition of fractals, the characterization of a form using its topological dimension (which is a whole number) proves its insufficiencies. That is why the notion – fractal dimension D_f (real number) was introduced.

The German mathematician Felix Hausdorff defines a new concept for the topological spaces, this way suggesting that the fractal dimension is proportional with the minimum number of spheres, of a give radius, needed for covering the measured object. To facilitate the computer work, the coverage is made with cubes instead of spheres.

Thus, for covering a curve of length 1, $N(s)=1/s$ cubes of side s are needed (fig. 1.3).

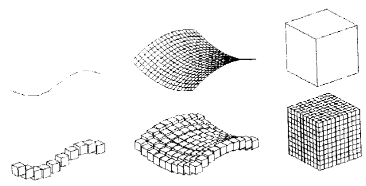


Fig. 3 Coverage of three Euclidian figures using cubes (equal sides).

For covering a surface of area 1, there are needed $N(s)=1/s^2$ cubes of side s and finally, to cover a cube of volume 1 – $N(s)=1/s^3$ cubes of side s and by induction the relation below is verified:

$$N(s) \sim 1/s^D,$$

where:

N(s) is the number of cubes of side s;

S is the scale coefficient or the length of the coverage cube's side;

D is the dimension of the object.

By applying logarithm to the relation above, we can deduce D

$$D \approx \frac{\log(N(s))}{\log(1/s)} \qquad (1)$$

Housedorff dimension is accepted as a good approximation for the fractal dimension D_f and according to this hypothesis, fractals are that forms with the Hausdorff dimension strictly greater than the topological dimension.

The fractal dimension is however difficult to calculate. There are some algorithms to calculate the fractal dimension and one of the easiest to implement is the box-counting algorithm.

2 Fractal dimension using"Box-counting" algorithm

In the specialized literature, there are many attempts to evaluate the fractal dimension. The algorithm based on the „box-counting" method offers two major advantages: it is easy to implement in case of using a computer and can be applied for images no matter how complex.

The "box-counting" fractal dimension, derived from the Hausdorff coverage dimension is given by the following approximation:

$$D \approx \frac{\log\big(N(s)\big)}{\log\big(1/s\big)} \qquad (2)$$

It is expected, that for a smaller s, the above approximation should be better,

$$D = \lim_{s \longrightarrow 0} \log \frac{N(s)}{\log(1/s)} \qquad (3)$$

If this limit exists, it is called the "box-counting" dimension of the measured object. Usually, this limit converges very slowly, that is why an alternative solution is used. Since the expression:

$$\log(N(s)) = D \cdot \log\left(\frac{1}{s}\right) \qquad (4)$$

is the equation of a straight line of slope D, the "log-log" curve described by the points of (log(N(s), log(1/s)) coordinates is plotted. Through linear regression (least squares method) the slope of the line that approximates the points' distribution is determined; this is the wanted fractal dimension.

Thus, the regression line has the form:

$$Y = a \cdot X + b \qquad (5)$$

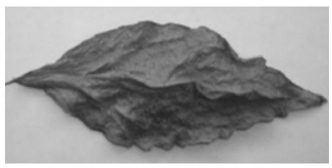
and the line slope(the value of the „**a**" coefficient), represents the fractal dimension.

The „box-counting" algorithm assumes to determine the fractal dimension according to the dependence of the object contour upon the used scale factor. It consists in successive image coverage with squares with equal sides (2, 4, 8, ..) and counting every time the squares that cover the object contour.

The points of coordinates (log(N(s)), log(1/s)), where s is the common side of the coverage squares, and N(s) the number of squares that contain any information, will be positioned approximately in a line and its slope will be the fractal dimension in "box-counting" context. In a synthetic representation, the algorithm for determining the "box-counting" dimension for binary images is the following:

1. the original image(binary) is read
2. the analysed region is selected
3. the box-counting dimension is calculated by counting each time the number of cubes N(s) that contain at least a point of the form. Logarithm is applied to the obtained values then, they are graphically represented by a curve with a slope that is the box-counting dimension.

For an example of how the algorithm is used, we'll consider the image of a leaf (fig. 4.1) from which we'll extract a binary version by neglecting all the pixels over a certain luminosity (fig. 4.2).



(1)

(2)

Fig. 4 (1) The initial image; (2) The binary (black-white) image version

Next, we'll apply the "box-counting" algorithm, described above, for different scale values s (only the squares that contain information are plotted, the ones that cover the leaf contour), using a software procedure, presented in detail in [2].

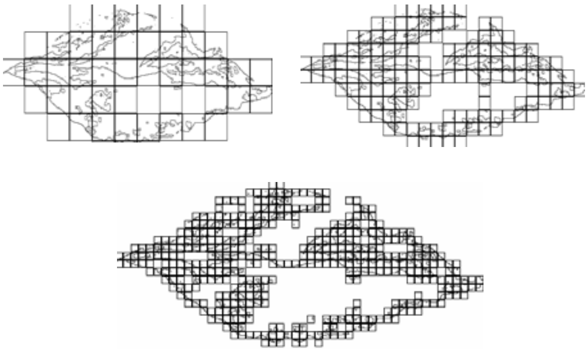


Fig. 5 Object coverage with squares of different side values –"s".

We obtain the values table and "log-log" curve from the figure below:

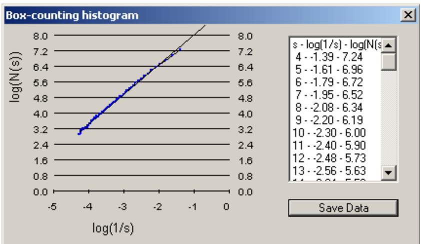


Fig. 6 The log-log curve and the

s, log(1/s),log(N(s)) values.

By using the least squares method, with the pairs of points (log(N(s)), log(1/s)), the regression line with the slope 1,55 is determined. Thus, the fractal dimension for the studied leaf is 1,55. In Romania, the fractal technique was applied, for the first time, in the domain of automatic processing of mammography by Daniela Crişan [2].

3 Fitofractals used in gentiana genus taxonomy

3.1 Experimental processing

The Gentianaceae family was divided as time passed, in many genera ([3], [4], [5]); from these the Menyanthes and Nymphoides genera came from the initial family and formed a new family named Menyanthaceae. At the moment, in the Romanian flora there are recognized 8 genera of the gentinacee's family: Blackstonia, Centaurium, Comastoma, Gentiana, Gentianella, Gentianopsis, Lomantogonium, Sweertia (V. Ciocârlan, 2000), obtained by separation of the central genus Gentiana that has 13 species, in Gentianella with 5 species and Gentianopsis with one species. In the reference paper [6], the Gentianopsis genus is included at Gentianella under the name Gentianella cilliata. Today, the Gentiana genus has 19 species spread all over our country. We mention that the separation of the Gentianella and Gentianopsis genera was made on pure morphological criteria ([5], [6]).

For establishing the independent position of the Gentianopsis genus, respective Gentianella ciliata for Gentiana genus, we have fractal analyzed the species Gentiana lutea and Gentianella ciliata. Taking into account that the fractal technique is used for the first time in botanic, for verifying its "sensibility", we have compared the fractal dimensions of the 2 taxons from Gentianaceae with one from Ranunculus genus from Ranunculaceae family.

The analysed material was acquired from the transversal section made through the root, stem and leaf of the mentioned taxoms. The extracted samples were analysed using fractal techniques based on the "box-counting" algorithm in order to use other criteria then the morphological ones for establishing the position of the Gentiana, Gentianella and Gentianopsis genera in the Gentianaceae family. Concretely, there were extracted window-images from the rind and the central cylinder of the root and stem and also from the mesophyll and leaf nervure/rib. The contours of the window-images were processed with the "box-counting" algorithm in order to establish the fractal dimensions for the analysed sections.

3.2 Results

For Ranunculus repens - root, the fractal dimension of the rind is D11=1.71 and the central cylinder dimension is D12=1.78:

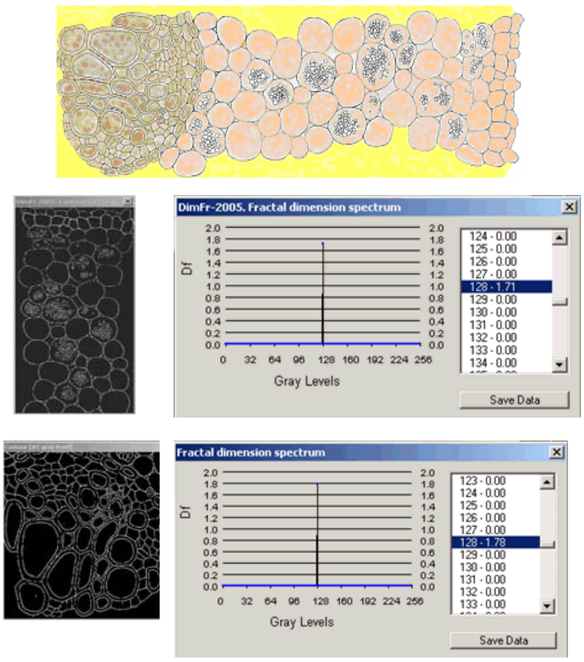


Fig. 7 Fractal dimensions for the root in the rind and central cylinder for Ranunculus repens.

For Ranunculus repens - stem, the fractal dimension of the rind is D21=1.72 and the dimension of the vascular bundle is D22=1.74:

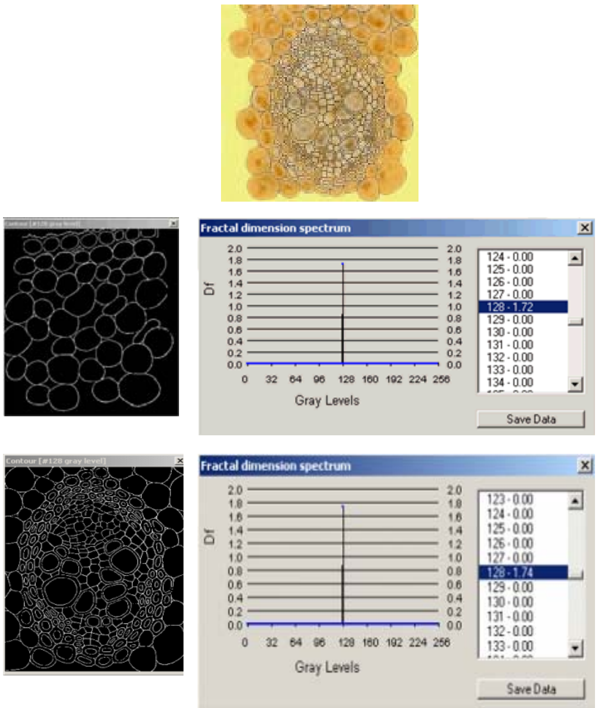


Fig. 8 Fractal dimensions for stem, in the rind and vascular bundle for Ranunculus repens.

For Ranunculus repens - leaf, the fractal dimension of the mesophyll is D31=1.51 and the dimension of the vascular bundle is D32=1.66:

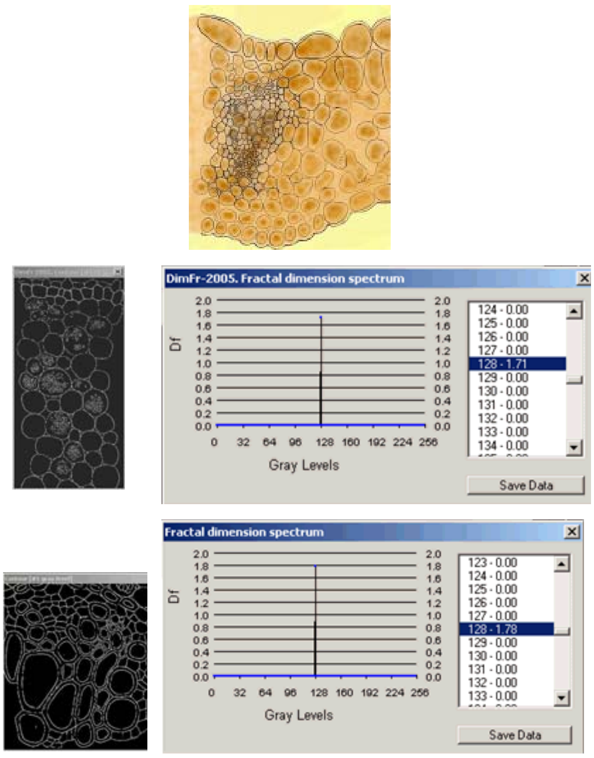


Fig. 9 Fractal dimension for the leaf, in mesophyll and vascular bundle at Ranunculus repens.

For Gentiana lutea - root, the fractal dimension of the rind is D11=1.76 and the central cylinder dimension is D12=1.80:

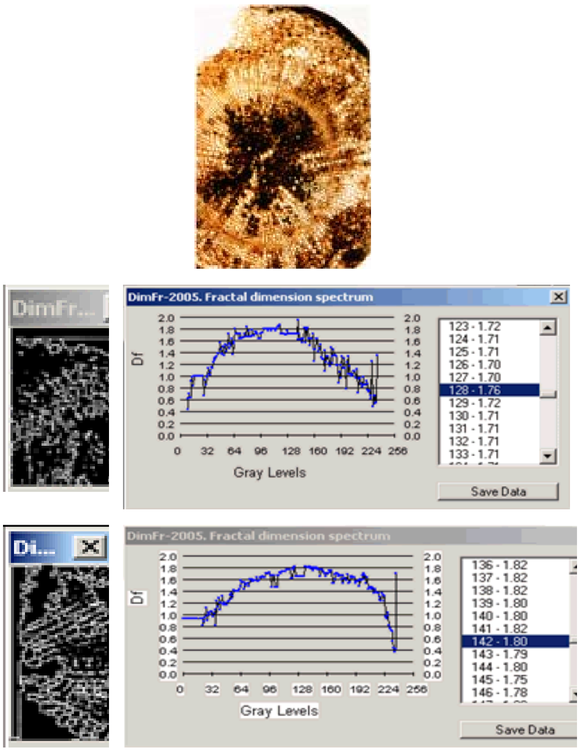


Fig. 10 Fractal dimensions of the root, in the rind and central cylinder at Gentiana lutea.

For Gentiana lutea - stem, the fractal dimension of the rind is 21=1.47 and the dimension of the central cylinder is D22=1.71:

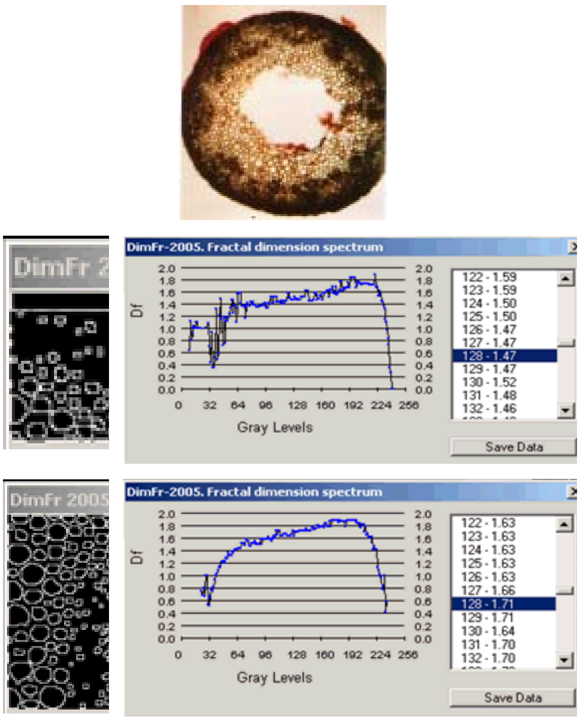


Fig. 11 Fractal dimensions for stem in the rind and central cylinder for Gentiana lutea.

For Gentiana lutea - leaf, the fractal dimension of the mesophyll is D31=1.64 and the dimension of the vascular bundle is D32=1.69:

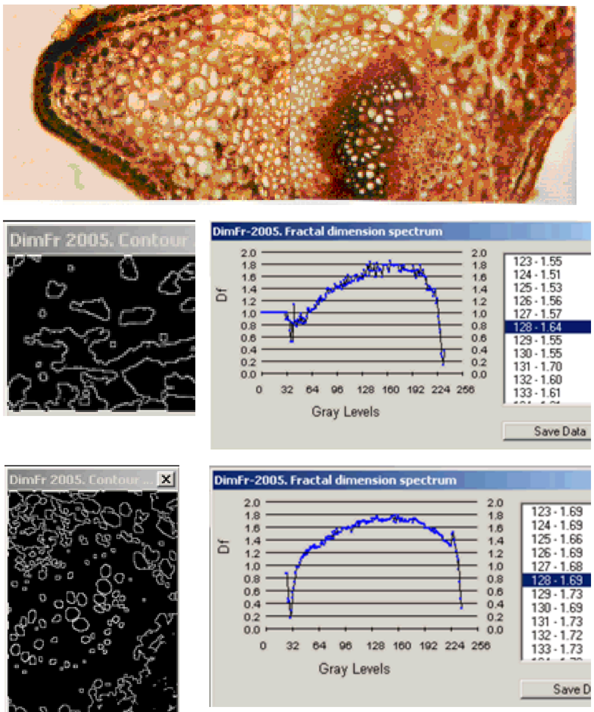


Fig. 12 Fractal dimensions for leaf, in mesophyll and vascular bundle for Gentiana lutea.

For Gentianella cilliata (Gentianopsis) - root, fractal dimension of the rind is D11=1.31 and the central cylinder dimension is D12=1.78:

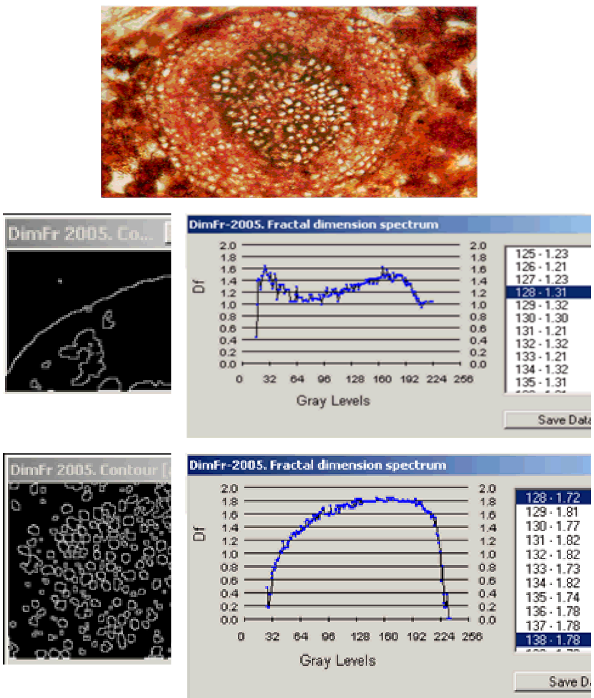


Fig. 13 Fractal dimensions for root, in the rind and central cylinder for Gentianella cilliata (Gentianopsis).

For Gentianella cilliata (Gentianopsis) - stem, the fractal dimension of the rind is D21=1.67 and the dimension of the central cylinder is D22=1.78:

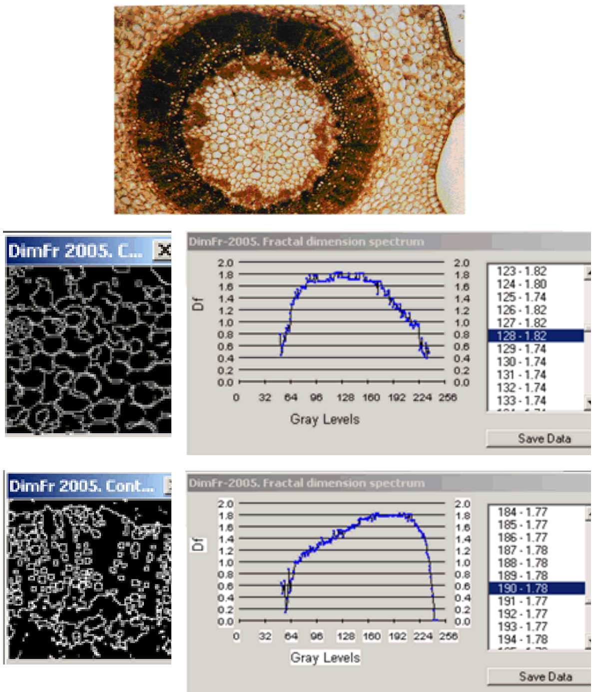


Fig. 14 Fractal dimensions for stem in the rind and central cylinder for Gentianella cilliata (Gentianopsis).

For *Gentianella cilliata (Gentianopsis)* - leaf, the fractal dimension of the mesophyll is D31=1.80 and the dimension of the vascular bundle is D32=1.81:

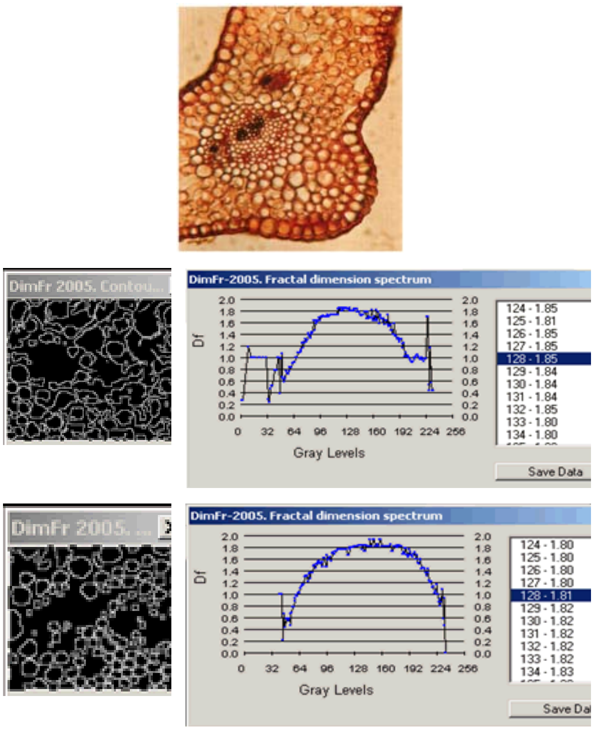


Fig. 15 Fractal dimensions for leaf, in the mesophyll and vascular bundle for Gentianella cilliata (Gentianopsis).

Tab. 1 Comparative table of the obtained data

D_f	root		stem		leaf	
	rind	central cylinder	rind	central cylinder	mesophyll	vascular bundle
Ranunculus repens	1,71	1,78	1,72	1,74	1,51	1,66
Gentiana lutea	1,76	1,80	1,47	1,71	1,64	1,69
Gentianella cilliata (Gentianopsis)	1,31	1,72	1.67	1,78	1,80	1,81

4 Conclusions

1. For leaf, at the 3 analysed genera, the fractal dimensions are close for the mesophyll and also for the vascular bundle; the different position of *Gentianela ciliata (Gentianopsis)* can be observed.

2. The authors present for the first time in Romania a new method to differentiate some species of the *Gentiana* genus, using the fractal analysis to establish the position of the *Gentianella* and *Gentianopsis* genera.

3. From our provisional observations it results that, from fractal dimensions point of view, the separation of the *Gentiana* and *Gentianopsis* genera is justified.

4. We consider that, by acquiring samples from more sections of a species (the studied one) for the statistical processing of the data, will lead to very precise characterizations of that species, for the first time in botanic.

5. The authors introduce for the first time in botanic the term fitofractal.

6. We consider that this paper opens a vast field in the botanic research domain and biology in general.

5. References

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