

# WIRES AND CABLES IN SOME DISCRETE STRUCTURES OF CIVIL ENGINEERING

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## Abstract

Wires and cables are valuable components for the design of mechanical structures, due to their high strength-to-lightness ratio. Their efficiency is only guaranteed when they are subjected to tensile stress: indeed, they exhibit a unilateral behaviour with a lack of stiffness in compression. The tensile state depends on the distribution of stress through the whole structure. The computation of this stress state requires to deal with nonsmooth relations arising from the modelling of the unilateral behaviours, leading to specific numerical strategies. Two examples of such structures are investigated. Tensegrity structures are light reticulated systems composed of bars in compression and a large number of tensioned cables. As poetically defined by Fuller [1], they are viewed as “islands of compression in an ocean of tension.” TexSol™ is a reinforced geomaterial made of sand and wires. This type of material is adapted for the embankments requiring a strong slope or works which may be subjected to a dilatation strain. In this case, the nonsmoothness is not reduced to the wire behaviour but concerns as well the frictional contact between grains and between grains and wires.

**Keywords:** Nonsmooth dynamics, unilateral behaviour, tensegrity, granular matter.

## Presenting Author's Biography

David Dureisseix. He entered the *Ecole Normale Supérieure de Cachan* (ENS Cachan, France) in 1988, and received the *Agrégation* in Mechanics in 1991. Its research activities began in the Mechanics and Technology Laboratory. He defended his Ph.D. thesis in 1997, and became Assistant Professor at the ENS Cachan. He is now Professor at the University Montpellier 2 in the Mechanics and Civil Engineering Laboratory. Head of the MultiContact System research team, its research activities concern domain decomposition, simulation of multiscale and multiphysics problems, and nonsmooth mechanics. Web page: <http://www.lmgc.univ-montp2.fr/dureisse>



## 1 Modelling of tensegrity structures

Tensegrity systems are innovative strut and cable systems used in Civil Engineering, Figure 1.

Motro [2] gives a precise but general definition: “Systems in a stable selfstress state including a discontinuous set of compressed components inside a continuum of tensioned components”. Consequently the unilateral behaviour of the cables only loadable in tension is a dominant feature through the whole structure. These structures require an initial prestress to exhibit stiffness.

The stiffness of the cables is often weaker than the one of the bars. It is then convenient to consider extensible cables with a unilateral behaviour, and elastic (or rigid) bars.

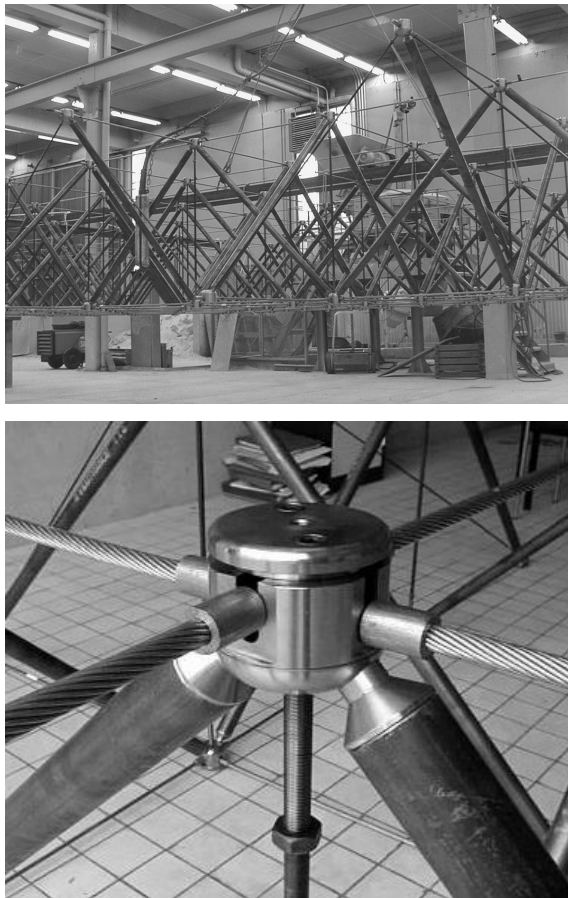


Fig. 1 A typical tensegrity structure and a node design, tensarch project (courtesy of R. Motro)

Under extreme loadings, cables may slacken, but contrary to buckling and plastification of a massive structure, a great amount of elastic energy may still be stored in a tensegrity structure. Consequently, with a sudden unloading, cables brutally get into tension again, which lead to impacts within the whole structure. The global behaviour therefore stands for a nonsmooth dynamical evolution.

There are several possible modellings:

- On one hand, one may consider that mass is concentrated at nodes; in such a case these nodes are rigid particles with their own dynamical evolutions, bars are elastic interactions between nodes, and cables are unilateral interactions with stiffness in tension and prestress; both are links between nodes.
- On the other hand, one may consider such structures as a discontinuous set of elastic bars, linked together mainly with cable-like interactions.

### 1.1 Dynamical evolution of an elementary component

For smooth motions, the dynamical equation involves the time-derivative of the velocities. Since shocks are expected, it is more convenient to write this equation as a *measure differential equation* [3, 4],

$$M dV + B^t \tau d\nu = F^d dt \quad (1)$$

where  $dt$  is a Lebesgue measure,  $dV$  is a differential measure representing the acceleration,  $d\nu$  a non-negative real measure relative to which  $dV$  happens to possess a density function.  $F^d$  is the external (smooth) prescribed force.

For the first modelling, (1) is the dynamical evolution of a node,  $M$  is the node mass,  $\tau$  is the tension of the bars, or in the cables, connected to the considered node.  $B$  is a node-to-link mapping matrix, containing the local basis to each connected link, and performing the assembly of external interactions.

For the second modelling, (1) is the dynamical evolution of a bar,  $M$  is the corresponding mass matrix,  $\tau$  is either the internal tension in the bar, or the external interaction with the connected cables.  $B$  is still a node-to-link mapping matrix.

When a large number of interactions are involved, a time stepping scheme is usually preferred to an event driven approach. Once a time discretisation is performed, an elementary time slab  $[t^-, t^+]$  of length  $h$  is considered. The variables evaluated at  $t^-$  (respectively  $t^+$ ) have the  $-$  (respectively  $+$ ) superscript. Since discontinuous velocities are expected, high order integration schemes are not necessary and even troublesome; first-order schemes are enough when many shocks may occur simultaneously.

We consider here the implicit Euler scheme underlining the impulsion  $\pi$  over the time step as the product of the time step  $h$  by an average tension  $\tau^+$  considered at the end of step. The dynamical equation (1) then reads:

$$MV^+ = MV^- + hF^d dt - B^t \pi \quad (2)$$

### 1.2 Dynamical behaviour of links between nodes

The static and elastic behaviour of a single bar indexed by  $\alpha$  is

$$\tau_\alpha = k_\alpha(e_\alpha + e_\alpha^0)$$

where  $k_\alpha$  is the stiffness of the bar,  $e_\alpha = B_\alpha u$  is the length variation of the bar,  $u$  is the total nodal displacement, and  $e_\alpha^0$  is the prestrain in the current bar.

The dynamical version of this behaviour relation involves the relative velocity  $\eta_\alpha = B_\alpha V^+$  and the impulsion  $\pi_\alpha = h\tau_\alpha^+$ ,

$$\pi_\alpha = h^2 k_\alpha [-\eta_\alpha + \frac{1}{h}(e_\alpha^- + e_\alpha^0)] \quad (3)$$

The modelling of a single extensible cable (also indexed by  $\alpha$ ) static behaviour takes the form of a piecewise linear function:

$$\tau_\alpha = \begin{cases} k_\alpha(e_\alpha + e_\alpha^0) & \text{if } e_\alpha + e_\alpha^0 > 0 \\ 0 & \text{if } e_\alpha + e_\alpha^0 \leq 0 \end{cases}$$

We can easily prove that this relation is equivalent to a complementary condition between the tension and a corrected length variation

$$\lambda_\alpha = -(e_\alpha + e_\alpha^0) + k_\alpha^{-1} \tau_\alpha \\ 0 \leq \lambda_\alpha \perp \tau_\alpha \geq 0 \quad (4)$$

(the orthogonality is here equivalent to set  $\lambda_\alpha \tau_\alpha = 0$ ).

According to the approach of [3], a dynamical discrete version of this behaviour is derived involving complementary conditions between relative velocity and impulsion:

$$\tau_\alpha^+ = \begin{cases} k_\alpha(e_\alpha^- - h\eta_\alpha + e_\alpha^0) & \text{if } e_\alpha^- - h\eta_\alpha + e_\alpha^0 > 0 \\ 0 & \text{if } e_\alpha^- - h\eta_\alpha + e_\alpha^0 \leq 0 \end{cases}$$

We can still recover a complementary formulation linking the impulsion to a corrected relative velocity

$$\lambda_\alpha^+ = \eta_\alpha + k_\alpha^{-1} h^{-2} \pi_\alpha - h^{-1}(e_\alpha^- + e_\alpha^0) \\ 0 \leq \lambda_\alpha^+ \perp \pi_\alpha \geq 0. \quad (5)$$

### 1.3 Solving strategies

The implicit nonsmooth problem to solve consists in:

- the dynamical evolution of the considered components of the structure (nodes with concentrated mass, or elastic bars) (2),
- the nonsmooth behaviours of the links (bars (3) or cables (5)),
- the boundary and initial conditions, and
- the time integration scheme to update the configuration at each time step.

Several solvers [5, 6, 7, 8, 9, 10] and software platforms can be used:

- When dealing with concentrated-mass nodes, approaches dedicated to granular media can be used, such as the Non Smooth Contact Dynamics approach [4], available in the LMGC90 software. This will be detail in next Section.

- With bars as elementary components, the solvers developed within the Siconos platform are relevant [11].
- When a domain decomposition is used, multilevel approaches such as in [12] can be more efficient.

### 1.4 Application to a tensegrity grid

A tensegrity grid is obtained with the duplication of a self stressed elementary module [13] with 12 cables and 4 bars (Figure 2), up to 256 modules.

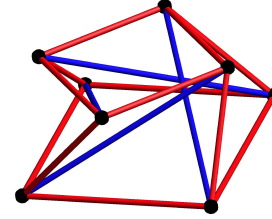


Fig. 2 Self stressed elementary tensegrity module

As boundary conditions, the lower nodes on two opposite edges are clamped, and a uniform vertical force field is prescribed on every node. We consider several loading amplitudes  $\alpha F_d$  with  $0 \leq \alpha \leq 1$ . The reader can find more details on the grid characteristics in [12], and in Table 1.

$H = 0.5$ m	Module height
$L = 1$ m	Module length
$S_c = 0.5 \cdot 10^{-4}$ m <sup>2</sup>	Cable section
$E_c = 10^{11}$ Pa	Cable Young modulus
$S_b = 2.8 \cdot 10^{-4}$ m <sup>2</sup>	Bar section
$E_b = 2 \cdot 10^{11}$ Pa	Bar Young modulus
$t_c^0 = 2000$ N	Lower cables prestress
$t_c^0 = \sqrt{2} \times 2000$ N	Upper cables prestress
$t_c^0 = \sqrt{(1 + 4 \frac{H^2}{L^2})} \times 2000$ N	Bracing cables prestress
$t_b^0 = -\sqrt{(5 + 4 \frac{H^2}{L^2})} \times 2000$ N	Bar prestress

Tab. 1 Characteristic parameters

Obviously, as  $\alpha$  increases, the stress redistribution is larger and larger: the number of slack cables increases (as well as the maximum value of internal tensions  $\tau_c$ ) to reach about 14 % of the whole set of cables when  $\alpha = 1$ . For this value of loading, the structure is still within its stable domain for which it still possesses a stiffness reserve. Such simulations are useful to check the integrity of such a structure under extreme loading conditions above normal service usage for which, in general, one assesses that no cable slackens; if this is the case, the strength of the structure could be endangered when the load decreases again and when slacken

cables suddenly reload: the rapid change in local apparent stiffness lead to dynamical loadings that can damage the nodes. Figure 3 shows the non linear evolution of this fraction of slack cables when the loading increases, Figure 4 and 5 shows the deformed structure (with an amplification coefficient of 10), and the tensions in cables.

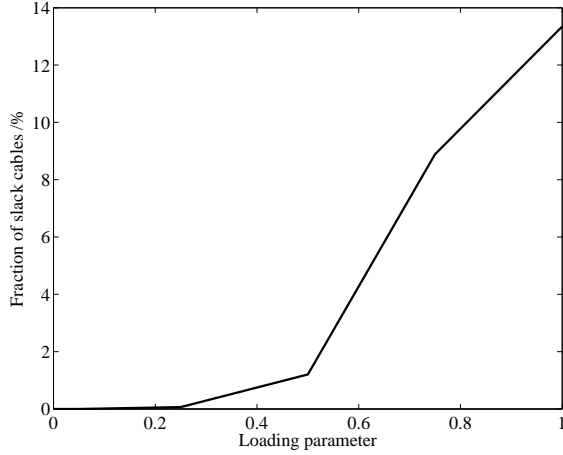


Fig. 3 Fraction of slack cables vs loading parameter  $\alpha$

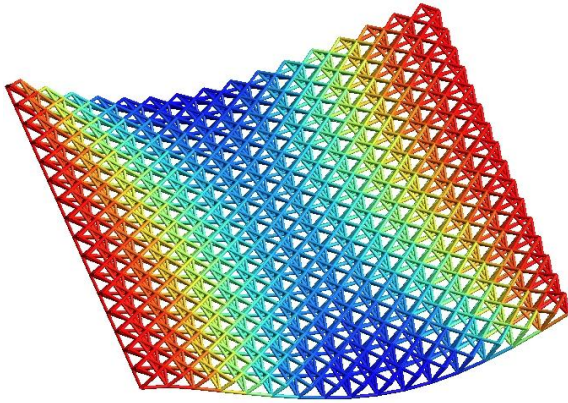


Fig. 4 Deformed structure (vertical displacement values), for  $\alpha = 1$

## 2 Modelling of wire-reinforced geomaterials

TexSol<sup>TM</sup> is a soil reinforcement process designed in 1984 by Leflaive, Khay and Blivet from the LCPC (*Laboratoire Central des Ponts et Chaussées*) [14, 15]. It is an original one because it mixes the soil (sand) with a wire. Although the wire volume is negligible compared to the sand one, the wire becomes a strong reinforcement when it tangles up randomly inside sand.

This type of material is adapted for the embankments requiring a strong slope or works which may be subjected to a dilatation strain (protection dome of a gas reserve for example). Indeed, the wire works in tensile

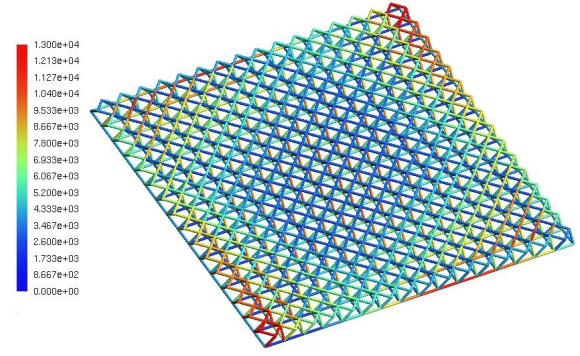


Fig. 5 Tensions in cables, for  $\alpha = 1$

directions and the wire network maintains the structure (when the wire density is big enough); the TexSol<sup>TM</sup> can be regarded as a composite material.

### 2.1 Non Smooth Contact Dynamics (NSCD) approach

TexSol<sup>TM</sup> being a reinforced granular material, the sand matrix can be integrated perfectly in a discrete modelling. On the other hand, the continuous nature of the wire requires a specific modelling effort.

NSCD is a discrete element method which simulates multibody vs. multicontact problems, privileging velocity fields [4]. On a single contact (indiced by  $\alpha$ ), NSCD evaluates the external forces and dynamic effects on the contactor point. To make such a transformation, we use  $B_\alpha^t$  and  $B_\alpha$  to move variables from the local contact frame to the global body frame and *vice-versa*. The local contact  $\alpha$  variables  $\eta_\alpha$  and  $\tau_\alpha$  (respectively the relative velocity and the contact reactions in the contact local frame) are defined with  $\eta_\alpha = B_\alpha V^+$  and  $R = B_\alpha^t \tau_\alpha$ . We also introduce the average impulsion  $\pi_\alpha$  as in previous Section, and we can write the problem as:

$$\begin{cases} \eta_\alpha - W_{\alpha\alpha}\pi_\alpha = \eta_\alpha^{\text{free}} + \sum_{\beta \neq \alpha} W_{\alpha\beta}\pi_\beta \\ \text{Law}[\eta_\alpha, \pi_\alpha] = \text{true} \end{cases} \quad (6)$$

The smooth dynamic effects are included in the expression of the relative free velocity  $\eta_\alpha^{\text{free}}$ . The Delassus operator

$$W_{\alpha\beta} = B_\alpha M^{-1} B_\beta^t$$

appears naturally in the dynamics reduced to contacts where  $M$  is the mass matrix. By this way, for a frictionless problem with a Signorini's contact condition, the system (6) reveals to be a standard Linear Complementary Problem (LCP) [16],

$$\begin{cases} \eta - W\pi = \eta^{\text{free}} \\ 0 \leq \eta \perp \pi \geq 0 \end{cases}$$

For a frictional contact problem, tangential reactions and tangential velocities have to verify similar non smooth relations.

## 2.2 Discrete modelling of the wire

In NSCD, a body is represented by its gravity center, its mass, its inertia moments and a set of contactors (*sphere, plane, polyhedron, point* ...). They describe the material boundary which is used by LMGC90 in the contact detection procedure which creates some contact elements between two contactors (*sphere-sphere* for example).

To discretize the wire, we split it into a collection of equidistant material points, with consistent masses. All these points must be connected by a behaviour law which accounts for a small segment of wire. The wire have to keep its free bending property and its unilaterality behaviour. Consequently, a wire contact law concerns only the normal direction and there is no constraint on the tangential directions. Then, we can introduce two unilateral laws:

- “rigid wire”, a unilateral law which can be described by

$$\eta_\alpha \leq h^{-1}(g_{\alpha,\text{ref}} - g_\alpha^-), \quad \pi_\alpha \leq 0, \quad \eta_\alpha \pi_\alpha = 0$$

where  $g_\alpha^-$  is the minimum distance between two contactors (i.e. the gap for the contact  $\alpha$ ) at the beginning of the considered time step;

- “elastic wire”, that includes unilaterality and the wire stiffness parameter  $k_\alpha$

$$\pi_\alpha = \begin{cases} 0 & \text{if } \eta_\alpha \leq h^{-1}(g_{\alpha,\text{ref}} - g_\alpha^-) \\ -h^2 k[\eta_\alpha - h^{-1}(g_{\alpha,\text{ref}} - g_\alpha^-)] & \text{otherwise} \end{cases}$$

A similar change in variables as in the previous Section leads to a similar LCP formulation.

## 2.3 Application to a numerical TexSol™ sample

This test consists in depositing a geometrically densified sample on a rubber plan (the rubber plan is assimilated to a collection of equal radius beads in a hexagonal distribution), Figure 6.

Figure 7 reports the pressure repartition on the ground. This graph confirms that the wire retains sand particles at the top of the sample, the more there are more wires. In the two reported cases, the slope friction angle of the TexSol  $\theta_t$  is higher than the sand one  $\theta_s$ ; their values correspond to experimental ones [14] lying between  $0^\circ$  to  $10^\circ$ . Moreover, this simulation emphasizes the paring arcs phenomenon of the granular assemblies described in [17].

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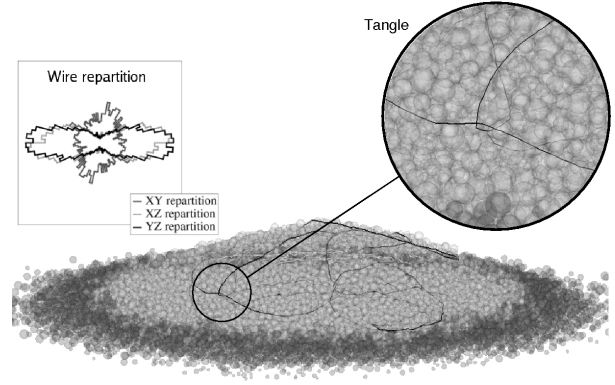


Fig. 6 TexSol™ sample after deposit on a rubber plan with the distribution of wire elements directions projected on several plans (( $O, x, y$ ), ( $O, x, z$ ) and ( $O, y, z$ )).

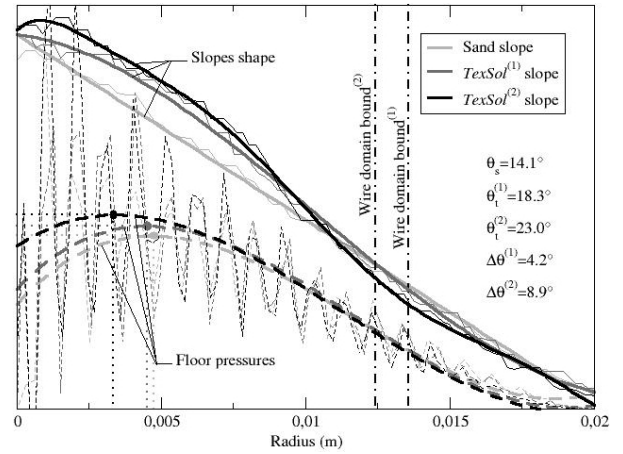


Fig. 7 Shape and pressure of sand, TexSol<sup>(1)</sup> with a wire volumic length is equal to 400 km.m<sup>-3</sup> and TexSol<sup>(2)</sup> with a wire volumic length is equal to 800 km.m<sup>-3</sup>.

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