AN EXTENDED PRICING AND INVENTORY CONTROL MODEL WITH REFERENCE EFFECTS

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Abstract

This paper addresses the simultaneous determination of pricing and inventory replenishment in the face of demand uncertainty with the objective of maximizing total expected discounted profits. We give a short review on recent work in literature: the operations-oriented stream of literature which uses rich cost models but rather neglect demand side dynamics and the marketing orientated stream which accounts for intertemporal demand correlations . In this paper we aim at a joint optimization of production and pricing decisions in inventory management, where demand is sensitive to the firm's pricing history and consumers do not only react to the current price. This socalled reference price - known from marketing literature - is formed on the basis of past purchases, where consumers e.g. buy products just because they are on sale and are less likely to buy a product after prices have gone up. Thus prices are perceived as discounts or surcharges relative to the reference price.
The investigated model is a combined stochastic dynamic pricing and inventory control model for a single item of a monopolistic firm based on periodic review. We study the optimal policies of pricing and ordering decisions and try to get some insight into the optimal solution's properties. We further investigate the impact of the newly introduced reference price on the optimal solution compared to optimal policies, using the commonly used but less realistic demand model of operations research. Since there are yet no analytical results available for this extended model the only means of providing a decision-support system for dynamic retail pricing and promotion planning is by numeric optimization via dynamic programming.

Keywords: Pricing, Inventory Control, Dynamic Programming

Presenting Author's Biography

Lisa Gimpl-Heersink. Master of Science in Technical Mathematics (OR, Statistics, Management Science) at Graz University of Technology. Working experience in life insurance mathematics and logistic software algorithm design. Currently PhD position as a research associate at the Vienna University of Economics and Business Administration, Institute for Production Management. Research interests: integrated demand and supply chain management, pricing and inventory control models (numerical and analytical optimization), dynamic programming.

1 Introduction

The profitability of a manufacturer/retailer selling frequently purchased consumer products or fashion-style-goods is strongly affected by ordering/production and pricing decisions. This paper adresses the problem of simultaneously determining a pricing and inventory replenishment strategy in the face of demand uncertainty. The commodity's price and ordering quantity are dynamically adjusted according to the prevailing inventory, the consumer's willingness to buy and the remaining length of the finite selling horizon. Not only are such models important in retail, where price dependent demand plays a significant role, but also in manufacturing environments in which production/ distribution decisions can be completed with pricing strategies in order to improve the firm's bottom line.

Traditionally pricing and replenishment strategies have been determined by entirely separate units of a company's organization, without proper mechanism to coordinate these two planning areas. Currently reengineering efforts are being initiated to systematically eliminate the organisational barriers between destinct functional areas within the same enterprise. Affected managers are faced with thousands of daily decisions and thus strongly depend on the support of corresponding decision support systems which cross traditional functional boundaries. This trend causes the need of advanced planning systems.

This paper addresses the important area in the interface between marketing and production/inventory planning and tries to fill the gap by developing an integrated inventory control and pricing model where the dynamics of both logistics and marketing are considered. There is no doubt that both methodologies are well developed and applied separately. Recent literature falls into two rather disconnected streams: The operations orientated stream and the marketing stream. Operations management usually uses rich cost models but rather simplistic demand models. Price optimization in marketing on the other hand, deals with setting optimal prices using rich and much more realistic and highly sophisticated demand models, underlaying a very simplistic cost model. In conclusion, both prevalent research streams consider only a partial picture of the relevant systems. This paper aims at filling the gap by developing an integrated inventory control and pricing model where the dynamics of both marketing and operations management are considered.

Many theoretical papers address the coordination of replenishment strategies and pricing policies in operations management. Just to name a few of them e.g. [1], [2] and [3] consider the newsvendor setting and e.g. [4], [5], [6], [7], [8], [9] a finite horizon multi-period model similar to ours and use safety stock levels as well as pricing decisions in order to hedge against demand uncertainties and maximize total expected profit. Each of them assumes a rather simplistic demand model with no temporal demand assumptions but non stationary (may vary over time) ordering costs proportional to the amount ordered. They find that under these assumptions a BSLP (base stock list price) policy is optimal. That is, in each period the optimal policy is characterized by an order-up-to level, referred to as a base-stock, and a price which depends on the initial inventory level at the beginning of the period. If the initial inventory level is below the base-stock level, an order is placed to raise the inventory level to the base-stock level. Otherwise, no order is placed and a discount price is offered. This discounted price is a nonincreasing function of the initial inventory.

Marketing literature is grounded on a market with repeated interactions where demand is sensitive to the firm's pricing history and thus accounts for intertemporal demand correlations. Here the aim is to assess optimal prices with respect to maximizing total expected profit, taking demand fulfillment for granted. Since consumers have a memory, the carrier of price is not only based on its absolute level, but rather on its deviation from some reference level resulting from the pricing history. As customers revisit the firm, they develop price expectations, which become a benchmark against which current prices are compared. A formulation that captures this effect is the so-called *reference price* which is a standard price against which consumers evaluate the actual prices of products they are considering. If the price of a brand is below its reference price, the observed price is lower than anticipated, resulting in a perceived gain. This would make the brand more attractive and raise demand (people buy products just because they are on sale). Similarly, the opposite situation would result in a perceived loss, reducing the probability that the brand is purchased (people are less likely to buy products after prices have gone up). An important consequence of this internal reference price formation is that although frequent price discounts may be beneficial in the short run, they may damage the brand in the long run when households get used to these discounts and reference prices drop. The reduced price becomes anticipated and loses its effectiveness, whereas the non-promoted price becomes unanticipated and would be perceived as a loss. E.g. [10], [11], [12], [13], [14] and [15] show that if this reference level is initially high, the firm will consistently price below this level, which has the effect of a skimming strategy. Similarly, a low initial reference level leads to a penetration type strategy.

In this work we want to combine the two features of the above discussed relevant literature streams: We want to use the rich and non-stationary cost models commonly used in operations research and include the richer and more realistic demand models, which account for intertemporal demand correlation and have been mainly applied by marketing people in the past. In this way we try to benefit from the dynamics of both prevalent models in literature and develop an integrated inventory control and pricing model.

2 The Model

In the following we analyze a single item, periodic review model. Nonnegative demands in consecutive periods are independent and their distributions depend on the item's price and consumer's reference price in accordance with general stochastic demand functions. Demand uncertainty can result in over- or underproduction, with resultant excess inventories or inability to meet consumer needs, respectively. Excess inventory incurs unnecessary holding costs, while the inability to meet consumer needs results in both loss of profit and potentially, the long term loss of customers, for which artificial penalty costs will be charged. Furthermore we assume that the company acts as a price setter or monopolist. Markets with competition could be analyzed only via a much more complex game-theoretic approach. The price charged and the inventory ordered in any given period can be specified dynamically as a function of the states of the system, depending on how much inventory is left from the last period and on the consumer's reference price, respectively. A replenishment order may and a pricing decision is to be placed at the beginning of each period. Stockouts are fully backlogged. Ordering costs are proportional to order sizes, while inventory carrying and stockout costs depend on the end-of-the period inventory level or shortfall. The objective is to maximize total expected (discounted) profits.

According to [16] we show that the evolution of demands over time can be represented by a tree-like structure shown in Fig. 1, if the demand distribution functions were discretized. Starting from each node several possible demand realizations can occur, expressed as branches stemming from that node. Assuming m possible next-period demand realizations at each node, the total number of scenarios will amount to m^T, where T is the number of periods considered in the selling horizon. At each period t each node is associated with the realisation of demand, the decision variables and the state variables. Complete enumeration would ammount to an exponential complexity of $O(m^T)$; therefore a stochastic dynamic programming approach with a compexity of $O(Tm)$ is described in the following to model the planning process as it reacts to demand realizations unfolding over time.

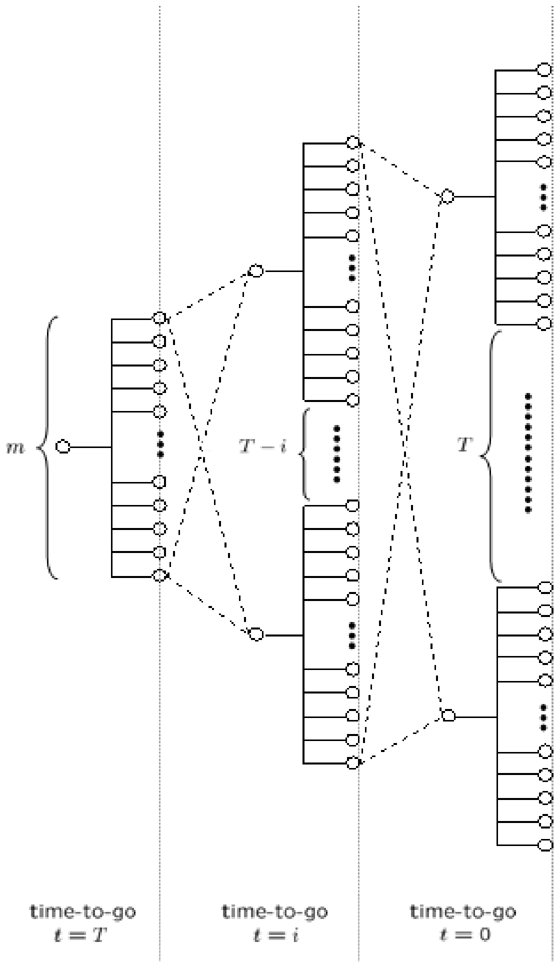

Fig. 1 Evolution of sample paths

2.1 The reference dependent demand model

The period's demand is defined additively as

$$D_t(p_t, r_t, \epsilon_t) = E[D_t(p_t, r_t, \epsilon_t)] + \epsilon_t, \qquad (1)$$

where ϵ_t follows a probability density function $f(\cdot)$ with mean $E[\epsilon_t] = 0$. $F(\cdot)$ denotes the corresponding probability distribution function. Demand is indexed by t to denote time dependence. In the following we will use the notation that t denotes the periods-to-go, thus $t = T$ denotes the beginning and $t = 0$ the end of the planning horizon, respectively. Reference price r_t is updated similarly as in [10], [11], [12], [14], [15] by simple exponential smoothing

$$r_t = \alpha(r_{t+1}) + (1-\alpha)(p_{t+1}), \quad 0 \le \alpha < 1, \qquad (2)$$

with r_t and p_t being reference price and observed price respectively for a brand in period t. This formulation was first introduced in the adaptive expectations framework by [17]. α is called the *memory parameter* and captures how strongly the reference price depends on past prices. Lower values of α represent a shorter term memory; in particular, if $\alpha = 0$, the reference price is the one-period lagged price for a brand ($r_t = p_{t+1}$) as in [18]. α also serves as a proxy for loyalty.

An in economic literature commonly used stochastic demand model is the piecewise linear demand function

$$D_t(p_t, r_t, \epsilon_t) = \beta_0 + \beta_1 \cdot p_t + \beta_2 \cdot \max\{p_t - r_t, 0\} + \beta_3 \cdot \min\{p_t - r_t, 0\} + \epsilon_t, \qquad (3)$$

with $\beta_0 \ge 0$ and $\beta_1, \beta_2, \beta_3 \le 0$ being estimated parameters such that the demand function is decreasing in price and increasing in reference price. The memory parameter α used in equation (2) is estimated in a way that we obtain the highest possible R^2 of equation (3) in OLS regression. Empirical studies in e.g. [11] and [12] find that estimated parameters of α range from $\alpha \in [0, 0.925]$.

[10] proposed that according to prospect theory, the effect of $(p_t - r_t)$ on demand is asymmetric, depending on whether it is positive or negative. Prospect theory predicts that when $(p_t - r_t)$ is negative, consumers perceive a gain; on the other hand, when it is positive, they

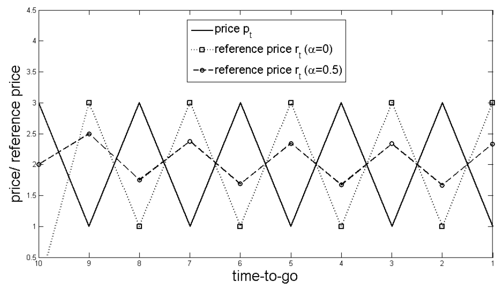

Fig. 2 Formation of reference price

perceive a loss. If equation (3) is symmetric with respect to the effect of gains and losses ($\beta_2 = \beta_3$), buyers are loss-neutral and the demand function is smooth. For loss-averse consumers the value function is steeper for losses than for gains ($\beta_2 < \beta_3$). In other words, a loss decreases value more than an equivalently sized gain would increase value. This is how we expect a rational consumer to behave.

2.2 The dynamic programming formulation

Let the state variables be x_t, the inventory on hand before ordering and r_t the consumers' reference price at the beginning of time period t. The decision variables y_t, the inventory level after ordering and p_t, the price charged at the beginning of period t effect the system's evolution. State and decision variables are related via the following transistion functions over time:

$$x_t = y_{t+1} - D_{t+1}(p_{t+1}, r_{t+1}, \epsilon_{t+1}) \qquad (4)$$
$$r_t = \alpha(r_{t+1}) + (1-\alpha)(p_{t+1})$$

Equation (4) gives the gross quantity of stock on hand at the beginning of period t, which equals the inventory on hand after ordering at the beginning of the previous time period less the total quantity actually sold during that period (compare figure 3). The objective is to maximize total expected profit over the entire selling horizon. Figure 4 gives a sketch of the system and the relationships among the variables and time.

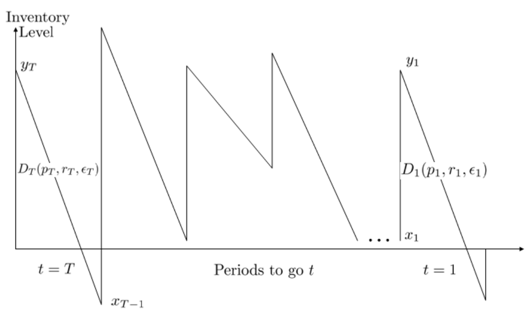

Fig. 3 Inventory sample path

For each period t we define c_t as the variable per unit purchase or production cost. Therefore the cost function is calculated as $c_t(y_t - x_t)$. At the end of the period t per unit inventory holding costs $h_t(u)$ are charged for each end-of-the period inventory level $u > 0$ and per unit backlogging costs $b_t(u)$ arise for a possible end-of-the period shortfall $u < 0$. Furthermore we assume in the following that the price charged and all costs arising are nonnegative ($p_t, c_t, h_t, b_t \geq 0$), that the price is never below the ordering costs ($p_t \geq c_t$), that the demand is nonnegative ($D_t(p_t, r_t, \epsilon) \geq 0$) and that $E[D_t(p_t, r_t, \epsilon)]$ is decreasing in p_t and increasing in r_t.

The single period profit is now given for period t by the following equation:

$$\begin{aligned}\pi_t(x_t, y_t, p_t, r_t, \epsilon_t) &= p_t \cdot D_t(p_t, r_t, \epsilon_t) - c_t(y_t - x_t) - \\ &- h_t \cdot \max((y_t - D_t(p_t, r_t, \epsilon_t)), 0) - \\ &- b_t \cdot \max((D_t(p_t, r_t, \epsilon_t) - y_t), 0).\end{aligned} \qquad (5)$$

At the end of the selling season ($t = 0$) there is the possibility that the leftover stock has some salvage value s_0, as it could perhaps be returned at some buyback price to the vendor or sold at a very low price (at or even below cost) in a liquidation sale. Possible shortfalls are reordered at the end of the last time period. Let $V_t^*(x_t, r_t)$ be the maximum expected profit from period t onwards (profit-to-go function), with initial inventory x_t and reference price r_t. For a specified discount factor γ the recursive bellman equation has the following form:

$$V_t^*(x_t, r_t) = \max_{y_t \geq x_t, p_t} \{J_t(x_t, y_t, p_t, r_t)\} \qquad (6)$$

$$\begin{aligned}J_t(x_t, y_t, p_t, r_t) &= E[\pi_t(x_t, y_t, p_t, r_t, \epsilon_t)] + \\ &+ \gamma E\left[V_{t-1}^*(y_t - D_t(p_t, r_t, \epsilon_t), \alpha r_t + (1-\alpha)p_t)\right],\end{aligned} \qquad (7)$$

with the boundary condition

$$V_0^*(x_0, r_0) = s_0 \cdot \max\{x_0, 0\} + c_0 \cdot \min\{x, 0\}, \ \forall x_0, r_0.$$

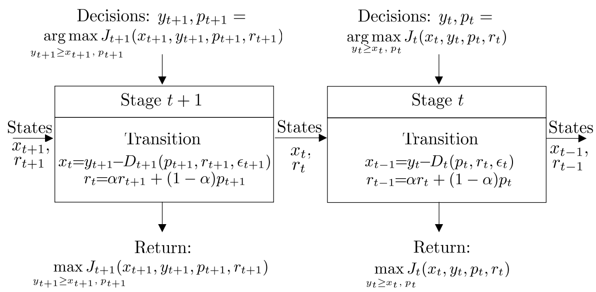

Fig. 4 Dynamic Program

Since only integer values of demand can be realized, we can discretize Eq. (7) by substituting expected values:

$$\begin{aligned} J_t(x_t, y_t, p_t, r_t) = & \sum_{D_{a,t}=0}^{\infty} p \cdot D_{a,t} - c_t(y_t - x_t) - \\ & - \sum_{D_{a,t}=0}^{\infty} h_t \cdot \max((x_t - D_{a,t}), 0) - \\ & - \sum_{D_{a,t}=0}^{\infty} b_t \cdot \max((D_{a,t} - x_t), 0) + \\ & + \gamma \sum_{D_{a,t}=0}^{\infty} \{V^*_{t+1}(y_t - D_t(p_t, r_t, \epsilon_t), \alpha r_t + (1-\alpha) p_t) \cdot \\ & \cdot P_{D_t}\left[D_t(p_t, r_t, \epsilon_t) = D_{a,t}\right]\}, \end{aligned} \tag{8}$$

where $D_{a,t}$ denotes the actual realised demand in period t and $P_{D_t}[\cdot]$ denotes the probability mass function of the number of actual sales in period t.

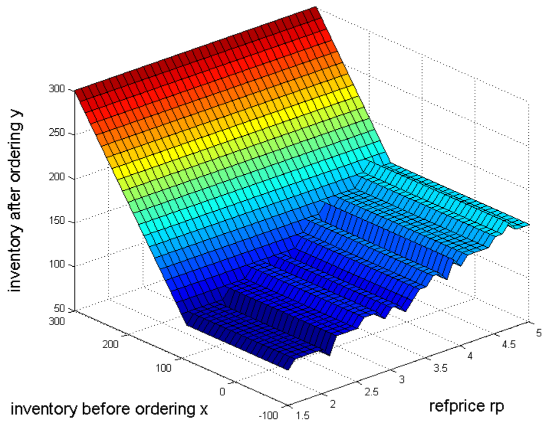

Fig. 5 The optimal ordering decision (lossneutral)

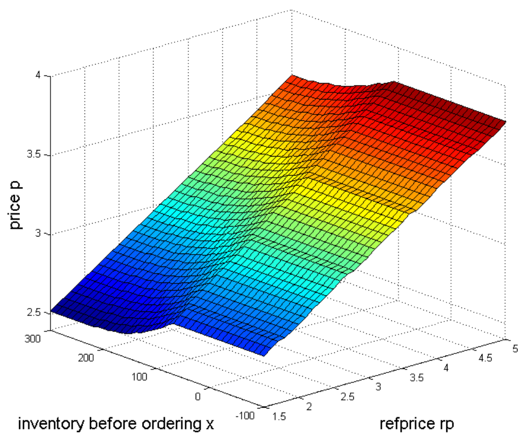

Fig. 6 The optimal pricing decision (lossneutral)

3 Numerical Study

In this section we report on a numerical study conducted to attain qualitative insights into the structure of optimal policies and their sensitivity with respect to several parameters. Due to the lack of data from a retailer in practice, a case study with real world data is left for future research. In order to obtain results with the highest possible managerial impact we basically use the linear demand model and the parameters [8] already obtained in an empirical study on a high-end women's apparel retailer, which we enrich by the dependence on reference price effects. In the interpretation of results we focus in particular on:

1. The sensitivity of the optimal base-stock/list-price combination with respect to different demand distributions.
2. The structure of the optimal inventory and pricing policy as a function of initial inventory before ordering and reference price.
3. The sensitivity of the optimal base-stock/list-price combination as a function of initial inventory before ordering and reference price.

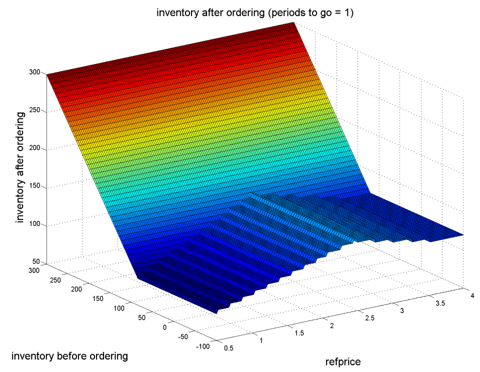

Fig. 7 The optimal ordering decision (lossaverse)

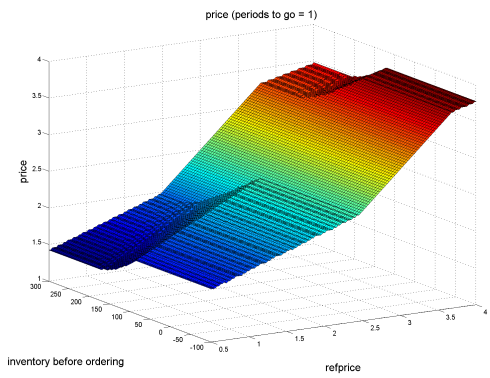

Fig. 8 The optimal pricing decision (lossaverse)

4. The sensitivity of the optimal base-stock/list-price combination with respect to the variability in demand.

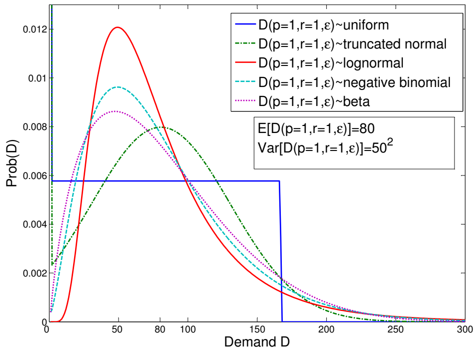

Fig. 9 Diverse probability density functions

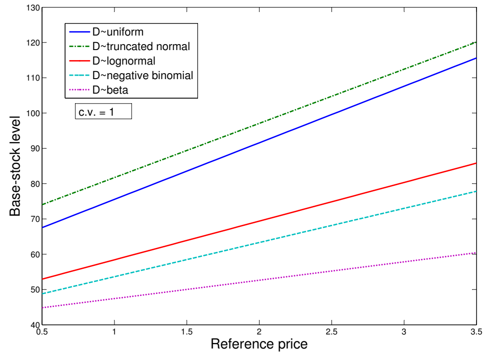

Fig. 11 Base-stock level, period-to-go = 1

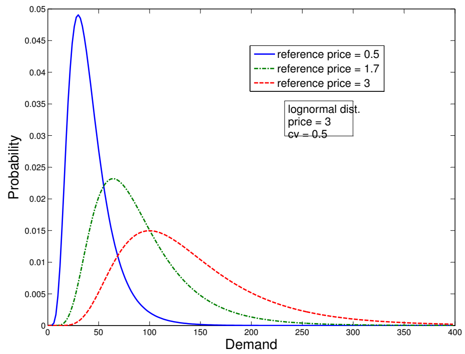

Fig. 10 Pdf dependending on mean demand

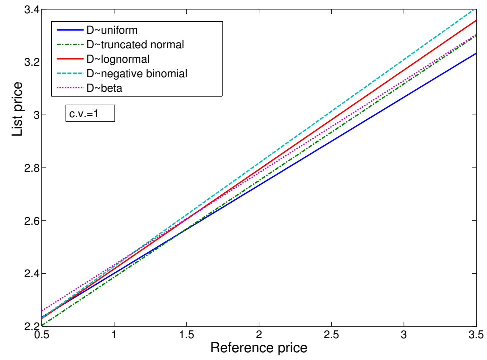

Fig. 12 List-price, period-to-go = 1

In our base scenario we use the demand model given in eq. (3) and set the parameters for lossneutral customer behaviour to $\beta_0 = 100, \beta_1 = -20, \beta_2 = \beta_3 = -40$ and $\beta_2 = -60, \beta_3 = -20$ for lossaverse customers, respectively. Because of the cost of capital, maintenance, insurance, loss, and damage, the per period holding cost rates amount to approximately 1% of the ordering costs ($c = 0.5, h = 0.005$). High service levels are ensured by setting the backlogging cost rates about the same magnitude as the ordering costs ($b = 0.4$). For an easier interpretation of the results, the salvage value is set equal to zero (s=0). For the same reason we assume that demand perturbation as all costs are stationary and do not vary over time, why we can omit the subscripts t. For the figures given in this paper no discounting factor is used ($\gamma = 1$). The stochastic term in the demand function follows an arbitrary distribution function with mean zero and variance cv $\cdot$ $E[D(p, r, \epsilon)]$, with cv denoting the coefficient of variation.

It is interesting that by adding refference effects to the demand model, a base-stock/list-price policy still turns out to be optimal (see fig. 5 and 6). Of course the optimal policies is now depending on the two states 'inventory before ordering' and 'reference price'. Thus we investigate if these policies have any new structural properties with respect to the reference price as a new system state. There is strong evidence that the optimal list-price is increasing in reference price (see fig. 6 and 8), which is intuitive, since given a higher reference price the retailer wants to skim as many margins as possible. He can charge a high price just below reference price without losing possible sales in order to raise his current profit and to keep the customers' reference price high for future periods. However, the situation is not so clear for the optimal inventory level. Simulations give that for lossneutral customers, the optimal inventory level is also increasing. This is because since for high reference price levels, a high mean demand is expected although a relatively high price is charged. Therefore the retailer wants to hedge against the higher variance cv$\cdot E[D(p, r, \epsilon)]$ by a higher inventory level (see fig. 5). In the case of lossaversion the situation behaves a litte differently. In fig. (8) one can observe a high slope of the optimal prices for the interval where price equals reference price ($p = r$). In this region an optimal inventory level is decreasing in reference price.

Fig. 9 illustrates the different shapes of the demand's probability density functions for several demand distributions (uniform, truncated normal,lognormal, negative binomial, and beta) with the same mean and variance. Note that the lognormal, negative binomial, and beta distribution has a considerably heavier tail

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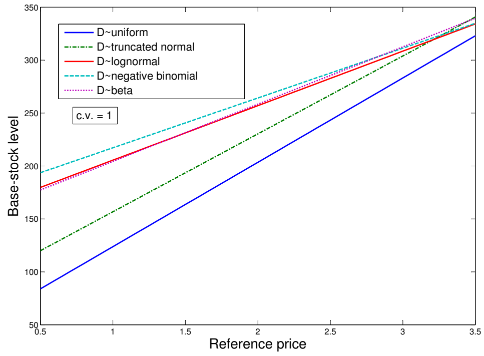

Fig. 13 Base-stock level, period-to-go = 10

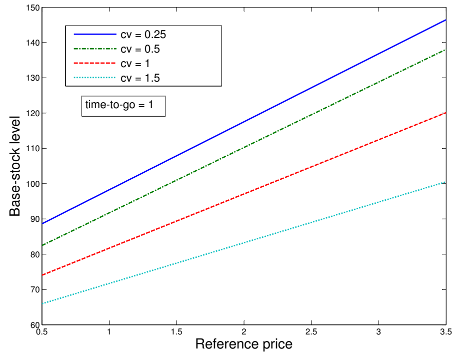

Fig. 15 Base-stock level depending on c.v.

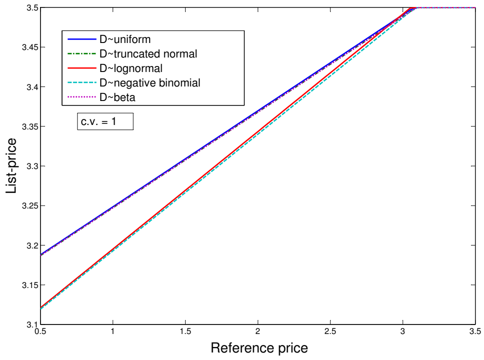

Fig. 14 List-price, period-to-go = 10

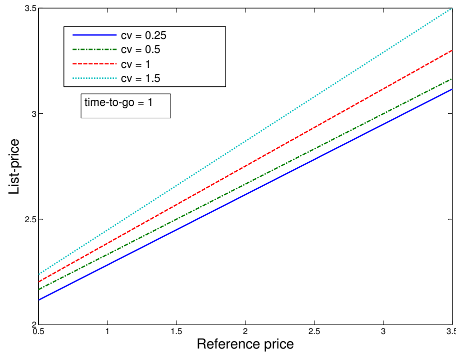

Fig. 16 List-price depending on c.v.

than the corresponding truncated normal and uniform distribution, respectively. All three of them are skewed to the left (their mode is smaller than the expected value) and allow only for positive demands, therefore there is no need for truncating negative demands as in the case of the normal distribution.

Since it is very costly to have unsold inventory on hand after the last time period, the main aim here is to reduce as much risk as possible of not selling the inventory on stock in the last time period. The higher degree of system uncertainty is - that is either a high coefficient of variation or a heavy tail distribution, the more the retailer aims for decreasing the standard deviation of demand. This can be obtained by reducing the mean demand, since then the standard deviation is reduced by the same proportion. Since demand is a decreasing function in price it is benificial to respond to an increase in system uncertainty by increasing prices (see fig. 12 and 16). This on the other hand results in a decreasing optimal base stock level (see fig. 11 and 15).

However in earlier time periods, the domitating objective is not to clear stock, but to optimize long run profits. In order not to run into expencive backlogging cost, the aim is to have sufficient inventory on stock. As we discussed above, it is clear that for a heavy tail distribution the risk of high demands is higher than for symmetric distribution function. Thus the optimal policy is to increase the inventory stock level for a higher degree of system uncertainty (see fig. 13), which on the other hand results in lower optimal prices (see fig. 14).

Fig. 17 shows that the optimal price is a decreasing function in inventory before ordering. Furthermore we can observe that the higher the reference price level is the later a discount on the listprice is given and the smaller the maximal discount is.

Fig. 18 and 19 show the difference in the optimal decisions, when we use a model with and without reference price effects. Where the corresponding intercepts in the base-stock and list-price is to be found, depends on the parameters and the range of the considered pricing interval.

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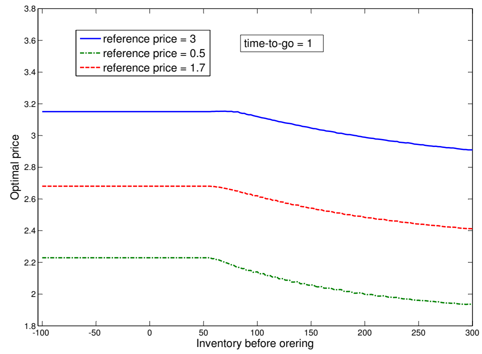

Fig. 17 Optimal prices depending on inventory

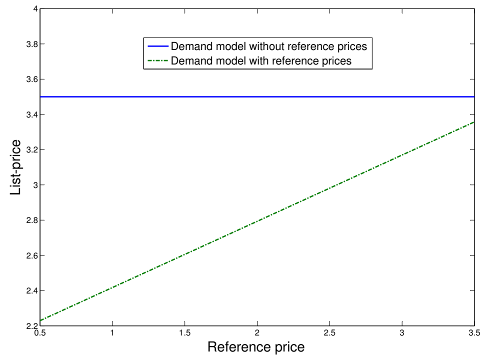

Fig. 19 List-price w/wo reference prices

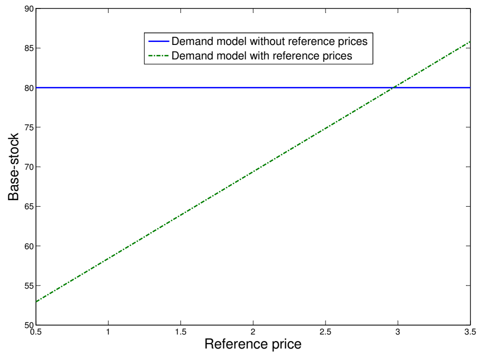

Fig. 18 Base-stock w/wo reference prices

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