FFT BASED COMPUTATIONAL MODEL FOR EM FIELD DEVELOPMENT ANALYSIS IN LASERS WITH ELECTRO-OPTICAL Q-SWITCH

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Abstract

Computational model, which treats EM field evolution inside an actively Q-switched laser, is presented in details. The EM field is represented by a 2D matrix and influence of the resonator elements on the EM field is described in five effective planes. The first plane represents out-coupling mirror, the second and the fourth plane laser rod ends, the third plane a center of the laser rod and the fifth plane the Q-switch element and the back resonator mirror. The matrix of the EM field is propagated between computational planes by use of a Fourier transform approach. The model is suitable for a treatment of a stable as well as for an unstable laser cavity and takes into account the lensing effect of the laser rod. Amplification of the laser light is described in terms of rate equations. Model can predict pulse energy, pulse temporal width, beam divergence, beam diameter and its intensity profile at different distances inside and outside a laser cavity. Computational time needed for a simulation of one pulse formation is of the order of minutes. Experiments performed on a ruby laser operating with a stable as well as with an unstable cavity configuration were compared to calculations and a good agreement was found.

Keywords: computer model, fast Fourier transform, laser cavity

Presenting Author's Biography

Dejan Škrabelj was born in Slovenia in 1981. In the year 2005 he has become B. Sc. in physics at the Faculty of Mathematics and Physics, University of Ljubljana. Since November 2005 he has been working at the Fotona laser company as an associate researcher and a Ph. D. student. He is cooperating with the department of Complex Matter at Jozef Stefan Institute, Slovenia.

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1 Introduction

A Q-switched laser produces short and energetic pulses and is thus found in different areas of modern world including scientific research, metrology and medicine.

Modern applications require Q-switched lasers operating in a stable as well as in unstable laser cavities. While both can produce powerful pulses the main distinction is found in the produced beam quality - unstable cavities output coupled with supergaussian mirrors can be nearly diffraction limited [1]. In addition, the intensity of the produced near field profile of such resonator can have "top-hat" shape found especially useful in medical applications, where beside a powerful pulse also a homogeneous illumination of a treating tissue is often required.

In a development phase of a new Q-switched laser system it is crucial to be able to predict laser pulse parameters such as pulse energy, pulse temporal width, beam divergence, beam diameter and its intensity profile at different distances inside and outside a laser cavity. The parameters like the distances inside of a laser cavity, mirrors' characteristics and characteristics of a laser rod can all affect the afore mentioned laser beam properties. In addition, in a real laser special care should be devoted to analyzing the influence of thermal lens introduced by a laser rod on a structure of a laser profile.

It is well-known that EM field inside an empty stable resonator takes the form of Hermite-Gaussian modes and Laguerre-Gaussian modes [2]. As a consequence, the properties of a laser beam produced by a stable laser resonator were often based on expansions in terms of empty resonator modes [3, 4]. The corresponding calculations usually take into account only the lowest order modes and it is often not known which set of the modes should be preferred. Foundation for the methods which avoid the mentioned drawbacks was set by Sziklas and Siegman, who introduced a method based on a fast Fourier transform algorithm (FFT) [5]. Despite the method being known since 1975 it has been used for a treatment of EM field formation inside of a laser cavity only a couple of times [6, 7]. It was found to be especially useful in the unstable cavity analysis, where diffraction effects inside the cavity play a major role in EM field formation process.

Marinček et al. [6] analyzed the spatio-temporal behavior of a free running erbium laser and a good agreement between calculations and experiment was found. Recently the model was rewritten to describe the development of the EM field inside a Q-switched laser and the main part of this paper will be focused on describing its main characteristics. At the end also a comparison between calculations and experiment will be presented.

2 DESCRIPTION OF THE MODEL

EM field of the light is in our model represented by $n \times n = 256 \times 256$ mesh points lying in the xy plane perpendicular to the resonator axis z. This particular plane is propagated within the resonator, which is described by five planes - the first plane represents an outcoupling mirror, the second and the fourth plane describe laser rod ends, the third a center of the laser rod and the fifth the back resonator mirror and the Q-switch element (Fig. 1). Values of the EM field at the mesh are calculated at each propagation between resonator planes because of a diffraction and in each resonator plane because of an influence of a certain resonator element.

Propagation between resonator planes located at coordinates z_1 and z_2 is described by the 2D fast Fourier transform technique. Fourier transform of the EM field in plane z gives its transversal spatial spectrum

$$e_t(s_x, s_y; z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(x,y,z) e^{2\pi i(s_x x + s_y y)}\, dxdy\,, \quad (1)$$

where s_x and s_y are spatial frequencies of the EM field $E(x,y,z)$. Contrary, if the spatial spectrum is known, the EM field in plane z is obtained by inverse Fourier transform

$$E(x,y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e_t(s_x, s_y; z) e^{-2\pi i(s_x x + s_y y)} ds_x ds_y \quad (2)$$

Propagating field must obey the amplitude wave equation

$$\nabla^2 E(\boldsymbol{r}) + k^2 E(\boldsymbol{r}) = 0, \quad (3)$$

where k represents the wavenumber. Now we put Eq. (2) into Eq. (3) and get an ordinary second order differential equation

$$\frac{d^2}{dz^2}\, e_t(s_x, s_y; z) + \left(\frac{2\pi}{\lambda}\right)^2 (1 - \lambda^2 s_x^2 - \lambda^2 s_y^2)\, e_t(s_x, s_y; z) = 0 \quad (4)$$

Equality $k = 2\pi/\lambda$, where λ is the wavelength of the light in a propagating medium, has been used. From

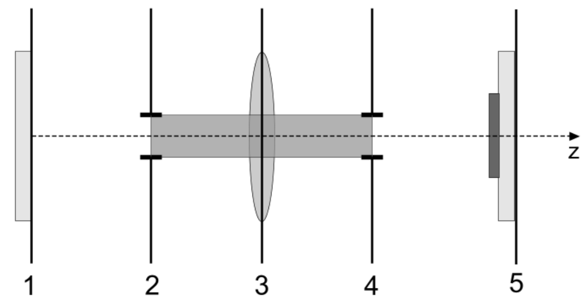

Fig. 1 Schematic drawing of the five effective planes of the laser resonator. Plane 1 represents the outcoupling mirror, planes $2-4$ describe the laser rod, and plane 5 describes a Q-switch element and the back resonator mirror.

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Eq. (4) finally the relation between transversal spatial spectra of the EM field in planes z_1 and z_2 is obtained

$$e_t(s_x, s_y; z_2) = e_t(s_x, s_y; z_1)\, e^{i\, \frac{2\pi}{\lambda} \sqrt{1-\lambda^2 s_x^2 - \lambda^2 s_y^2}\, d}\,, \tag{5}$$

where $d = z_2 - z_1$. To summarize: for a propagation of the EM field for a distance d, the following operations have to be performed: Fourier transform of the EM field at plane z_1 (number of operations is $\propto n^2$), propagation of the transversal spectrum for a distance d achieved by the multiplication with the exponential term ($\propto n^2$) and finally the inverse Fourier transform ($\propto n^2$) gives the propagated EM field. The number of operations required to calculate the diffraction pattern by use of other methods, as is direct evaluation of the diffraction integral, is greater than n^2.

Laser cavity is bounded with two mirrors (planes 1 and 5 in the model). A laser mirror changes not only an amplitude of an incident EM field (E_{in}) defined by its reflectance $R(x, y)$, but also its phase. Reflected EM field (E_{ref}) has the form

$$E_{ref}(x, y) = E_{in}(x, y)\, R(x, y)\, e^{\frac{-ik(x^2+y^2)}{r}}\,, \tag{6}$$

where r is the inner radius of curvature of the mirror. Back resonator mirror reflects all the radiation back into the cavity, so its reflectance is unity and it only changes the phase of the incident EM field. On the other side of the resonator the outcoupling mirror reflects only a certain part of the radiation back into the cavity, while other part is transmitted. Reflected part of the EM field has the form of Eq. (6), whereas the transmitted part (E_{tr}) is determined by

$$E_{tr}(x, y) = E_{in}(x, y)\, (1 - R(x, y))\, e^{\frac{-ik(x^2+y^2)}{2f}}\,. \tag{7}$$

Because the transmitted EM field passes the outcoupling mirror the phase change is given with its focal length f (exponential term in Eq. (7)), which is characterized with both radii of curvatures (inner and outer) and the mirror's refractive index. The outcoupling mirror can have a non-uniform reflectance profile $R(x, y)$, e.g. a supergaussian profile, found to be very useful in an unstable laser cavity.
From the computational point of view reflectance and the exponential terms in Eqs. (6, 7) are $n \times n$ matrices and therefore the number of operations required to reflect or transmit the matrix representing the EM field through a mirror is $\propto n^2$.

In the model the laser rod is presented with planes $2-4$. Amplification of the EM field based on a stimulated emission takes place in the third plane and is described in terms of rate equations, which couple the photon density Φ and the population inversion density n of the laser medium and are for the Q-switched operating regime given as [8]

$$\frac{\partial \Phi}{\partial t} = (\sigma c n \frac{l'}{l} - \frac{\Lambda}{t_R})\Phi\,, \tag{8}$$

$$\frac{\partial n}{\partial t} = -\sigma c \Phi \gamma n\,, \tag{9}$$

where σ is the stimulated emission cross section, c is the speed of light in the laser rod, l' is the length of the laser rod, l is the length of the resonator, Λ presents the internal resonator losses and $t_R = 2l/c$ is the round-trip time inside of the resonator. The factor γ is equal to $1 + g_2/g_1$ with g_2/g_1 being the ratio between the upper and the lower laser level degeneracies. The photon density and the amplitude of the EM field are related with $|E| \propto \sqrt{\Phi}$. The phase of the incident field is preserved during the amplification.
In a typical Q-switch cycle population inversion builds up before a Q-switch element becomes transparent. This is the reason why the initial population inversion is the model input parameter. Usually a homogeneous distribution $n(x, y) = const.$ is assumed. Initial elements of the EM field are generated as complex numbers with randomly distributed phases describing the spontaneous emission, which is the origin of the laser action.
The population inversion is built up by the absorption of photons with appropriate frequencies to pump atoms of a laser medium from the ground to the pump levels. Energy of other emitted photons is transferred to phonons resulting in a heating of the laser rod. Thermal load causes the temperature gradient and stress inside the rod and as a consequence a parabolical profile of the refractive index inside the rod is obtained. Because of the variation of optical length with radius photons see a thermal lens when they pass the rod. For this reason, in the model a lens is placed in the third resonator plane. Elements constituting the plane of the EM field are multiplied with $e^{\frac{-ik(x^2+y^2)}{2f_t}}$ whenever they pass the laser rod. f_t is thermally induced focal length inversely proportional to the heat flow per unit volume in the laser rod [8], which means that stronger lensing is expected when laser pulses are produced with higher repetition rates.
In the planes 2 and 4 the points of the EM field plane lying within the circle characterized with the laser rod radius remain unaffected, while the outer points are smoothly set to 0 to account for the diaphragm effect of the laser rod.
The population inversion and thermal lens are $n \times n$ matrices.

The voltage driven Q-switch element mediates the resonator losses - at first the voltage is applied and all photons are reflected out of the resonator. However, when the population inversion inside the laser rod is built up, the voltage is switched to 0 and the Q-switch element becomes transparent so the pulse formation begins. The Q-switch element is in the model described with a time - dependent function, which considers a realistic transit between high and low losses level.

In the simulation at first matrices describing the influence of the resonator elements are constructed. Then the matrix of the EM field, which is initially filled with complex numbers with random phases, is sequentially propagated with the use of Fourier transform between the resonator elements where the EM field is calculated. The time step in the model is equal to the half time of flight through the resonator t_F. For a 30 cm long

resonator it is approximately $\sim 1\,ns$. For a typical Q-switched pulse width $\sim 10\,ns$ (value is known from experiment) it would mean that matrix passes the central plane only for ten times. To decrease the time step there are actually two matrices describing the EM field in the model. They are propagated in opposite directions and it seems that they are a good compromise between the time efficiency and the computational accuracy of our model. Time needed to simulate a formation of a Q-switch pulse is on the order of minutes.
Propagation of the matrices is repeated until the upper laser level becomes depleted. At the times $t_j = jt_F$ $(j = 1, 2, 3, ...)$ part of the EM field leaks out of the resonator. The overall pulse intensity I is the sum of all individual contributions transmitted at the times t_j, $I(x, y) = \sum_j I(x, y, t_j)$. The effective pulse width is estimated as

$$\Delta t = \frac{\sum_{j=0}^{\infty} I(t_j)\, t_F}{\sum_{j=0}^{\infty} I(t_j)}\,, \tag{10}$$

where $I(t_j)$ represents the intensity summed over all matrix elements at the time t_j. From the overall intensity of the generated pulse the beam diameter in the x direction is determined as [9]

$$r_x = 2\sqrt{\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x, y)x^2\,dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x, y)\,dxdy}}. \tag{11}$$

Beam width in the y direction is obtained if x is replaced with y. Of course, in the discrete space double integration translates to double summation. The far field intensity distribution is a Fourier transform of the near field and from it the divergence of the beam can be obtained [9].

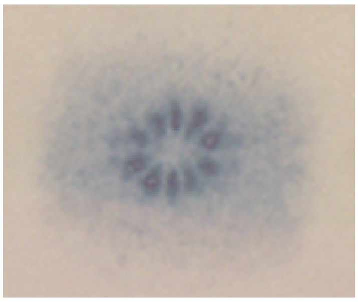
3 COMPARISON WITH EXPERIMENTS

Predictions of the model were compared to experiments performed on a ruby laser operating with a stable as well as with an unstable laser cavity. Ruby system is under active investigation at the moment and is planned to become a new Fotona medicine laser.

In the ruby crystal the upper laser level is split into two sublevels, namely $2\bar{A}$ and $\bar{E}$. Both levels are doubly degenerated and separated for $29\,\mathrm{cm}^{-1}$. The laser action occurs only between the $\bar{E}$ and the ${}^4A_2$ ground level, which is 4 times degenerated. The relaxation process between the upper two levels is very short, i.e. of the order of ns. This is the reason why both levels can be considered as a one 4 times degenerated level, as a temporal width of pulse is expected to be long compared to $1\,\mathrm{ns}$. This is the case in the ruby laser, where one can adopt $\gamma = 2$.

The parameters of a ruby laser used in our simulations are as follows: $\sigma = 2.5{\cdot}10^{-20}\,\mathrm{cm}^2$ and fluorescent lifetime of the level $\bar{E}$ $\tau = 3\,\mathrm{ms}$. We observed no significant differences in the NF profile structure depending on the ruby crystal axis orientation with respect to the

A)



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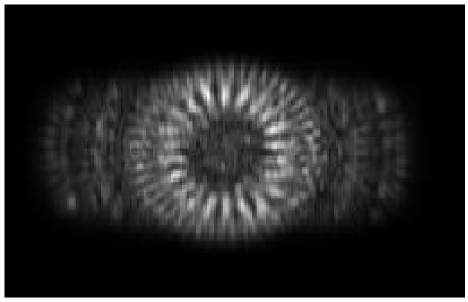

Fig. 2 NF profile of the ruby system operating in a stable cavity (A) and profile obtained with the model by setting T=0.99 (B).

polarizing element, which is a part of the QS element. Consequently all present simulations were performed with the refractive index of 1.763.

First experiment was performed with a stable laser cavity outcoupled with a classical mirror with reflectance of 40%. The ruby laser rod inserted into $38\,\mathrm{cm}$ long cavity was pumped with two flashlamps contained in the close-coupled reflector geometry. Standard Kodak thermal paper placed behind the outcoupling mirror was used to record NF intensity profiles. Areas of a profile, which contain greater amount of the power, burn darker spots into the thermal paper.

Measured near field is presented in Fig. 2, A). The non-circular shape is assigned to non-uniform pumping of the laser rod. Profile contains pronounced $\mathrm{TEM}_{0,10}$ pattern. Such near field is not an usual output of a stable plano-concave cavity thus we anticipated that its origin lies in reflections from coatings on the rod end surfaces resulting in an additional resonator formation. It could be formed between the front resonator mirror (plane 1) and the laser rod end surface on the back mirror resonator side (plane 4) or between the back resonator mirror (plane 5) and the rod surface on the front resonator mirror side (plane 2).

Parameter T defining the ratio between transmitted and incident photon flux on a laser rod end surface was introduced in the model in order to investigate the influence of the additional resonator. On an edge plane main part of the EM field is transmitted and further propagated to the resonator mirror and back to the rod, while other part is reflected in an edge plane. An elliptical shape of the initial population inversion was assumed in order to take into account a non-circular shape of the

profile.
Profile shown in Fig. 2, B obtained by setting $T = 0.99$ confirmed that the internal resonator was indeed formed between planes 1 and 4. In calculated patterns the power scale is reversed. Greater amount of power is represented by brighter spots.

Finally the supergaussian mirror was mounted into the resonator. In order to minimize the lensing effect of the laser rod the initial tests of the constructed system were performed with single-shot laser pulses. The experimentally observed NF profile is shown in Fig. 3. One can see that its edge is more intense than its center. Pronounced intense rings are followed with less intense ones when moving toward the center of the profile, where intensity again increases. Contrary to the stable resonator in case of the unstable configuration outcoupled with the supergaussian mirror the profile becomes cylindrically symmetrical. Further tests of our

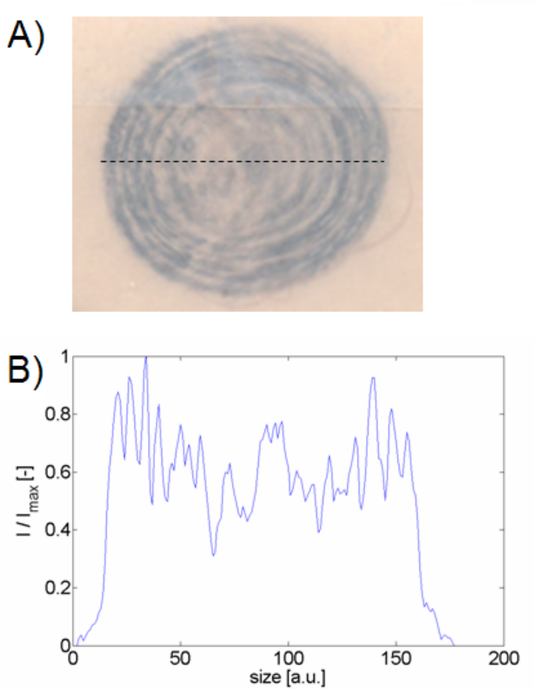

Fig. 3 Measured NF profile of the ruby system operating with the unstable cavity (A) and its cross-section (B).

system were performed also with higher repetition rate pulses connected with greater heat accumulation inside of the rod, which resulted in a stronger lensing behavior. During the initial tests an optical damage in the laser rod occurred.

The behavior of the ruby laser at different thermal loads of the flashlamp radiation in the laser rod was studied also with the model. The NF profiles and their cross-sections calculated with $f_{rod} = 40\,\mathrm{m}$, $f_{rod} = 30\,\mathrm{m}$ and $f_{rod} = 20\,\mathrm{m}$ are shown in Fig. 4. One can see NF profile generated using $f_{rod} = 40\,\mathrm{m}$ gives a good agreement with the experimental result (see Fig. 3).

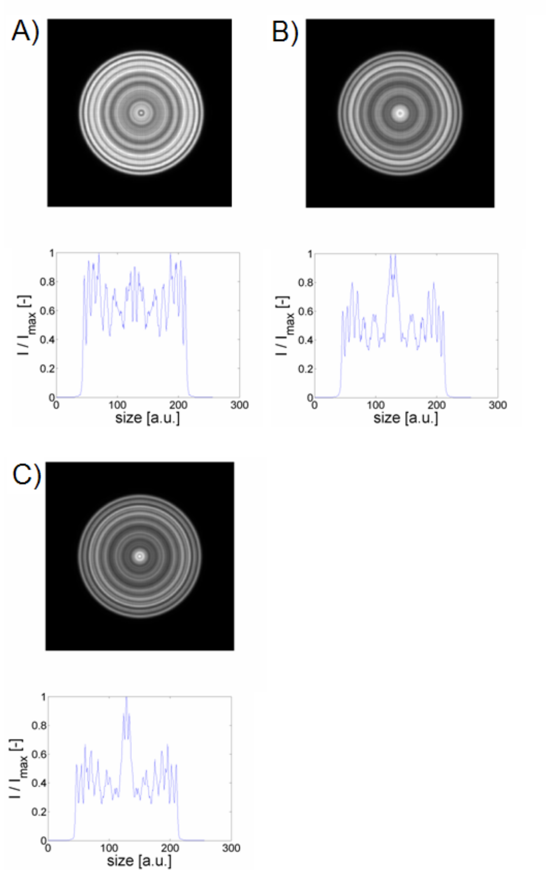

Fig. 4 Transverse field patterns and their cross-sections obtained with numerical simulations using $f_{rod} = 40\,\mathrm{m}$ (A), $f_{rod} = 30\,\mathrm{m}$ (B) and $f_{rod} = 20\,\mathrm{m}$ (C).

When the focal length of the laser rod decreases to $f_{rod} = 30\,\mathrm{m}$ an intensity peak appears in the center of the NF profile and becomes even more pronounced when $f_{rod} = 20\,\mathrm{m}$. In addition, in the later case central maxima in intensity profiles occur in the resonator planes, which explain the observed optical damage inside of the laser rod.

The model was tested also in the temporal domain. With fast Si diode we measured the shape of the pulse produced by the constructed unstable ruby system and compared it to the calculated one. In order to make comparison as accurate as possible, both pulses were selected to have the same amount of energy of approximately $1\,\mathrm{J}$.
From Fig. 5 it is evident that the simulation quite correctly describes time development of the photon flux. The measured FWHM of the pulse is $25\,\mathrm{ns}$, while the FWHM obtained from simulations is $27\,\mathrm{ns}$. Estimation provided by Eq. (10) is $\Delta t = 29\,\mathrm{ns}$. An about $10\,\%$ disagreement between the calculated and measured FWHM confirms the model also in the temporal domain.

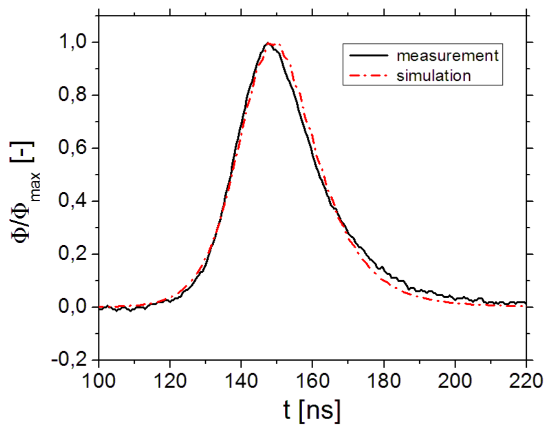

Fig. 5 Measured and calculated temporal shape of the output pulse from a Q-switched ruby laser.

4 CONCLUSION

We have designed the model for a treatment of a Q-switched laser. The development of the EM field inside the laser cavity is described with two matrices propagated in opposite directions through a laser resonator being described by five computational planes, where an influence of a specific laser component is considered. A propagation between planes is performed with the Fourier transform approach. One of the advantages of the model is short computational time for one-pulse-formation cycle being of the order of minutes.

Despite of the simplicity of the model, the measurements of the spatio-temporal properties performed on the experimental ruby laser operating with the stable laser cavity and the unstable laser cavity confirmed the computations.

One of the aims of our group is to develop a ruby laser that produces a top-hat near field intensity profile. In order to obtain a better top-hat profile, further optimization will be carried out taking into account computational results.

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