MULTIOBJECTIVE PREDICTIVE CONTROL BASED ON FUZZY HYBRID MODELLING

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Abstract

In this paper, a multiobjective predictive control based on fuzzy hybrid modeling and solved by Evolutionary algorithm is presented. At every instant, a genetic method is used to find the Pareto optimal front. Provided that only one input can be applied to the system, different criteria are used to explore ways of using Pareto Optimal front. Besides, EMO solution allows obtaining a new tuning method for the weighting factor of the typical Model Predictive Control. An illustrative experiment on a hybrid tank system is conducted to show the benefits of the proposed approach.

Keywords: Predictive Control, Hybrid Predictive Control, Fuzzy Model, Evolutionary Multiobjetive Optimization.

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1 Introduction

Evolutionary Multiobjective Optimization (EMO) has been applied for a large number of static problems. For example, an EMO solution has been designed for a static assignment problem with fuzzy rules [1]. On the other hand, some works have been developed for dynamic multiobjective problems and there is a lack of methods that allow testing them adequately [2].

Recently, there are some works related to Multiobjective Predictive Control problems. One important application of Dynamic Multiobjective is the Multiobjective Predictive Control proposed by Kerrigan and Maciejowski [3]. They solve the multiobjective predictive control problem based on prioritized constraints and objectives. In this case, the most important optimization problem is solved first and the solution to this problem is then used to impose additional constraints on the second optimization, etc. Also, the control action of predictive controller proposed is solved based convex programming techniques by considering certain convexity assumptions. Thus, prioritized multiobjective predictive controller can be solved on-line without redesigning the controller off-line, however, this increase in flexibility also demands an increase in the amount of on-line computational power.

Núñez et al. [4] presents a comparison of different multiobjective predictive controllers applied to an olive oil mill. A typical Model Predictive Control (MPC) approach based on mono-objective function, a prioritized multiobjective predictive controller and structured MPC controller are compared. The last one, structured MPC, uses a decision list to select the current objective function must be supplied to the MPC control action. Based on simulation tests, the prioritized multiobjective predictive controller gives the best results without the need of tuning weights as the typical MPC, however complex software is required and therefore, a big computational cost is needed. An intermediate solution is the structured MPC however abrupt behavior in the switching between different objectives is observed.

Zambrano and Camacho [5] describe a multiobjective predictive control algorithm based on a goal attainment method, which considers the different objective functions as constraints for the minimization of relaxation variables. This multiobjective predictive controller allows the specification of different goals, like economic factor, at different operation points and it was applied to a solar refrigeration plant and formulated for variable configuration systems. The results show benefits of including the multiobjective approach.

Recently, Subbu et al. [6] present a multi-predictive multi-objective optimization approach for thermal power plants. In this case, the approach integrates an adaptive predictive model based on neural network, optimization based on multi-objective evolutionary algorithms and decision making methods based on Pareto frontier techniques. In this case, the multiobjective optimization approach gives the Pareto front of set-points for the MPC controllers, as a supervisory level and the selection of the current set-points applied is based on a decision making.

Thus, multiobjective predictive controllers, reported at the specialized literature, are interesting developments, however the multiobjective optimization problem is not completed solved, and sub-optimal solutions are generated. In this work, we propose a dynamic multiobjective predictive controller based on fuzzy hybrid modeling that provide solutions using evolutionary algorithms and also based on that to generate an on-line tuning method for the typical MPC used at industrial processes.

There are different approaches of hybrid predictive control design. Slupphaug [7] and Slupphaug & Foss [8] describe a predictive controller with continuous and integer input variables solved by nonlinear mixed integer programming.

Bemporad & Morari [9] present a predictive control scheme for hybrid systems solved by using Mixed Integer Quadratic Programming (MIQP). The main problem of the MIQP is its computational complexity which increases the time to find the solution. To overcome this problem, Thomas et al. [10] propose partitioning the state space domain.

On the other hand, Potočnik et al. [11] propose a hybrid predictive control algorithm with discrete input based on reachability analysis. The computation time is reduced by building and pruning an evolution tree.

Núñez et al. [12] present a hybrid predictive control strategy based on a fuzzy model. The key element of the fuzzy identification is the detection and estimation of switching regions by combining fuzzy clustering and principal component analysis. The nonlinear NP-Hard optimization problem was solved efficiently by the use of genetic algorithms in terms of accuracy and computational time. Thus, a typical Model Predictive Control objective function was considered where the reference tracking and control effort were minimized.

In this work, we propose the use of evolutionary multiobjective optimization (EMO) in order to provide better solutions for Multiobjective MPC based on Fuzzy Hybrid modeling, that are not explored with the typical MPC.

The outline of the paper is as follows. In Section 2 Hybrid Fuzzy Predictive Control based on Evolutionary Multiobjective Optimization is presented. Section 3 gives simulation results of the control of a hybrid tank system. Finally, in Section 4 the conclusions are included.

2 Hybrid Fuzzy Predictive Control based on Evolutionary Multiobjective Optimization

2.1 Hybrid Fuzzy Predictive Control (HFPC)

The HFPC strategy is a generalization of model predictive control (MPC), where the prediction model based on fuzzy logic includes both discrete/integer and continuous variables. In general, HFPC minimizes the following objective function:

$$\min_{\{u(k),u(k+1),\dots,u(k+N_u-1)\}} J = J_1 + J_2$$
$$J_1 = \sum_{j=N_1}^{N_y} \left(\hat{y}(k+j) - r(k+j)\right)^2 \qquad (1)$$
$$J_2 = \sum_{j=N_1}^{N_u} \lambda(k+j)\,\Delta u(k+j-1)^2$$

where J is the objective function, $\hat{y}(k+j)$ corresponds to the j step-ahead prediction of the controlled variable based on a fuzzy model, $r(k+j)$ is the reference, $\Delta u(k+j-1)$ is the increment of the control action, and $\lambda(k+j)$ is the weighting factor. N_1, N_y and N_u are the prediction horizons and the control horizon, respectively. The optimization results in a control sequence being $\{u(k),\dots,u(k+N_u-1)\}$.

Note that the weighting factor is important for stability purposes. However, finding the optimal weighting function sequences is not an easy task. Therefore, a fixed weighting is commonly used [4].

As we assume that the hybrid predictive control problem includes discrete input variables, the optimization could be solved by evaluating all possible feasible solutions, Branch & Bound and other algorithms like GA [13].

2.2 Hybrid Fuzzy Predictive Control based on Evolutionary Multiobjective Optimization (HPFC-EMO)

The HFPC-EMO strategy is a generalization of HFPC, where control objectives are similar to HFPC but the optimal control action must be chosen based on a criterion that selects a solution from the Pareto Optimal region of following problem:

$$\min_{\{u(k),u(k+1),\dots,u(k+N_u-1)\}} \{J_1, J_2\}$$
$$J_1 = \sum_{j=N_1}^{N_y} \left(\hat{y}(k+j) - r(k+j)\right)^2 \qquad (2)$$
$$J_2 = \sum_{j=N_1}^{N_u} \Delta u(k+j-1)^2$$

where J_1, J_2 are the objective functions to minimize. The optimization solution is a control sequence region called the Pareto Optimal set. To formalize the previous notions, the following concepts are important to define.

Let us consider $U^i = \{u^i(k),\dots,u^i(k+N_u-1)\}$ a control action sequence, where $u^i(k+t)$ belongs to the set of feasible control action.

A solution U^i Pareto-dominates to a solution U^j if and only if,

$\left(J_1(U^i) \le J_1(U^j) \wedge J_2(U^i) < J_2(U^j)\right)$ or $\left(J_2(U^i) \le J_2(U^j) \wedge J_1(U^i) < J_1(U^j)\right)$.

A solution U^i is said to be Pareto optimal if and only if there is not U^j that Pareto-dominates U^i. Pareto optimal set P_s contains all Pareto optimal solutions. The set of all objective function values corresponding to the solutions in P_s is $P_F = \left\{\left(J_1(U^i), J_2(U^i)\right) : U^i \in P_s\right\}$. P_F is known as Pareto optimal front. If the discrete manipulated variable case is considered, where the feasible input set is finite, the size of P_s is also finite.

As just one input *u(k)* has to be applied to the system, a criterion is used in order to find the best control sequence that belongs to the Pareto front $U^i = \{u(k),\dots,u(k+N_u-1)\}$. This criterion will be related to tracking error as well as control effort and it will be defined later.

The multiobjective optimization could be solved by evaluating all solutions, Branch & Bound and other algorithms [14]. Next, an efficient optimizer based on Genetic Algorithms (GA) is described for this problem.

As the Pareto front could be hard to obtain in real time application, new method that connects HFPC-EMO solution with HFPC is suggested. With an off-line model, a HPFC-EMO is used to obtain the responses of the system. Based on the dynamic Pareto Optimal front, the weight value λ at instant k could be estimated, that connects HFPC-EMO solution with the HFPC. Then in the real-time application the estimated weighting function $\lambda(k)$ from HFPC-EMO could be used instead of a fixed value. This also could be interpreted as a new tuning method for the weighting factor of typical MPC.

2.3 Optimization based on Genetic Algorithm

Genetic algorithm is used to solve the multiobjective optimization problem because it can efficiently cope with mixed-integer non-linear problems. The idea is to find the Pareto optimal set and then from the Pareto optimal front that will be used to obtain the control action. A potential solution of the GA is called individual. The individual can be represented by a set of parameters related to the genes of a chromosome and can be described in a binary or integer form. The individual represents a possible control-action sequence $\{u(k),...,u(k+N_u-1)\}$, where each element is a gene, and the individual length corresponds to the control horizon N_u.

Using genetic evolution, the fittest chromosomes are selected to assure the best offspring. The best parent genes are selected, mixed and recombined for the production of an offspring in the next generation. For the recombination of genetic population, two fundamental operators are used: crossover and mutation. For the crossover mechanism, the portions of two chromosomes are exchanged with a certain probability in order to produce the offspring. The mutation operator alters each portion randomly with a certain probability [14].

In order to find the Pareto Optimal set, the best individuals are the ones that belong to the best Pareto Optimal set found until current iteration (due to the fact that there are solutions that belong to the Pareto Optimal set but they are not found yet). Solutions that belong to the best Pareto Optimal set will have a fitness function equal to 0.9 and the other solution fitness will be equal to 0.1 in order to hold the solution diversity.

The genetic algorithm approach in HFPC-EMO provides a sub-optimal Pareto front very close to the optimal one. The tuning parameters of the GA method are the number of individuals, number of generations, crossover probability, mutation probability and the stopping criteria.

Once the best Pareto front is found, different criteria could be applied in order to select the best control action at every instant.

In this work we propose the following criteria:

- To choose the control action solution from the Pareto front that has a minimal tracking error value.
- To fix a bounded tracking error and to choose the control action solution from the Pareto front that satisfies that tolerance and has a minimal control effort.

2.4 Relation between HPFC-EMO and HPFC

Once the Pareto Optimal front is obtained as a function of instant *k* (dynamic front), the equivalent HFPC problem is obtained by identifying the weighting factor *λ(k)*. Provided that an analytical solution of the Pareto front is not available, two methods are proposed in order to estimate the *λ(k)* factor:

A) LS Method.- By non-linear regression or least mean square, to estimate an analytical function of the Pareto front using non-linear regression. After that, at the optimal solution chosen from Pareto front, the slope of this analytic function is obtained and it is related with the *λ(k)* factor.

B) IM Method.- In this case, firstly a range of possible *λ(k)*is determinated considering if (J_1*, J_2*) is the selected Pareto front point, the following inequalities have to be satisfied:

$$\lambda \geq 0\,,\ \forall (J_1, J_2) \in P_F\,, J_1 + \lambda J_2 \geq J_1* + \lambda J_2* \qquad (3)$$

After that, *λ(k)* is equal to the minimum λ that satisfies equation (3).

Once *λ(k)* is obtained by using one of these two alternatives and registered for a time period. After that, a model for *λ(k)* could be identified and will provide a tuning method at every instant for a HFPC. Thus, we propose a conventional HFPC with λ tuned from the multiobjective problem (HFPC-EMO).

3 Non-linear System

3.1 Process Description

The tank system is shown in Figure 1. The controlled variable in this case is the level in the first tank h_1, and the manipulated variable is the voltage of the pump in the inlet (u), which has discrete levels. It is also assumed that both levels h_1 and h_2, are measured.

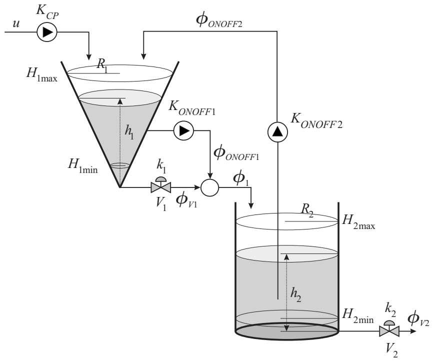

Fig. 1 The tank system plant.

The behavior of the system is defined by the following nonlinear differential equations and algebraic equations, which define the switching regions:

$$\frac{dh_1}{dt}\cdot\pi\cdot\frac{R_1^2}{H_1^2}h_1^2 = K_{CP}\cdot u + \phi_{ONOFF2} - \overbrace{V_1 h_1}^{\phi_{V1}} - \phi_{ONOFF1}$$

$$\frac{dh_2}{dt}\cdot\pi\cdot R_2^2 = \overbrace{V_1 h_1}^{\phi_{V1}} + \phi_{ONOFF1} - \overbrace{V_2 h_2}^{\phi_{V2}} - \phi_{ONOFF2} \qquad (4)$$

$$if\ (h_2 \geq H_{2\min}) \text{ and } (h_1 < H_{1\max})\ then\ \phi_{ONOFF2} = K_{ONOFF2}$$

$$if\ (h_1 \geq H_{1\max}) \text{ and } (h_2 < H_{2\max})\ then\ \phi_{ONOFF1} = K_{ONOFF1}$$

where h_1 and h_2 stand for the level of the liquid in the first and the second tank and $H_{1\min}, H_{1\max}, H_{2\min}, H_{2\max}$ stand for switching levels. Note that the rules in (4) represent the switching or hybrid behavior. Based on input/output data and the identification method proposed by Núñez et al [12] the structure of the fuzzy model is defined as:

$$x_{k+1} = \sum_{i=1}^{s\cdot R_i} \mu_i(z_k)\left(a_i x_k + b_i u_k + r_i\right)$$

$$\mu_i(z_k) = \frac{\prod_{r=1}^{p} B_{i,r}(z_{k,r})}{\sum_{i=1}^{s\cdot R_i}\prod_{r=1}^{p} B_{i,r}(z_{k,r})} \qquad (5)$$

where the variable in premise is $z_k = h_{1,k}$ whose membership functions are shown in Fig 2 and the consequent vector equals $x_c = \left[h_{1,k}\ u_k\ 1\right]^T$. The parameters of the fuzzy model ($\theta_i = \left[a_i\ b_i\ r_i\right]$), are obtained by least means squares.

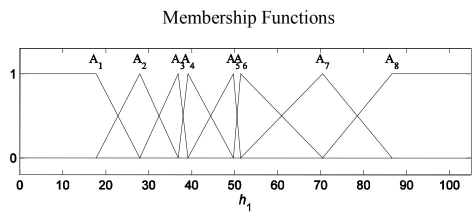

Fig. 2. The corresponding membership functions of the fuzzy model.

3.2 Hybrid Fuzzy Predictive Control based on Evolutionary Multiobjective Optimization (HFPC-EMO)

The tuning parameters of the multiobjective function in (2) are given by $N_1 = 1$, $N = N_y = N_u = 3$. For the optimization based on GA the mutation probability equals 0.001, the crossover probability equals 0.7, the generations number equals 50, the individuals number equals 30 and maximum number of generations is used as stopping criterion. The proposed controllers in section 2.2 will be compared with a conventional HFPC with $\lambda = 0.001$ reported by Núñez et al [12].

HFPC-EMO is tested using the criteria defined in section 2.3:

HFPC-EMO1. To choose the solution from the Pareto front that has a minimal tracking error value.

HFPC-EMO2. To fix a bounded tracking error equal to 0.5[cm] and to choose the control action from the Pareto front that satisfies that tolerance and has a minimal control effort.

HFPC-EMO3. To fix a bounded tracking error equal to 1[cm] and to choose the control action from the Pareto front that satisfies that tolerance and has a minimal control effort.

Fig. 3 and Fig. 4 show the controlled variable (conic tank level h_1) and the manipulated variable (discrete voltage of pump u), respectively for criteria 1, 2, 3 and HFPC with $\lambda = 0.001$.

Fig 5 and Fig 6 show the controlled and the manipulated variables detailed in the range of 1100 to 2000 s.

From figures 3 to 6 and as we expected from the criteria definitions, HFPC-EMO satisfies each criterion applied to the controlled variable and the control effort is reduced as the tracking error increases. The conventional HFPC has a bigger control effort than HFPC-EMO2 and HFPC-EMO3, but its response follows the reference in a better way. HFPC-EMO1 reaches the lower tracking error, but its control effort is the biggest.

In Table 1 the mean values and standard deviation of tracking error and control effort are shown for data of figures 3 and 4 (performance with a fixed reference). From Table 1, HFPC-EMO3 reaches the lowest control effort, but the biggest tracking error as we can observe also from figures 5 and 6. Therefore, Table 1 shows that the solutions of the different criteria belong to a Pareto front, which is shown in Figure 7.

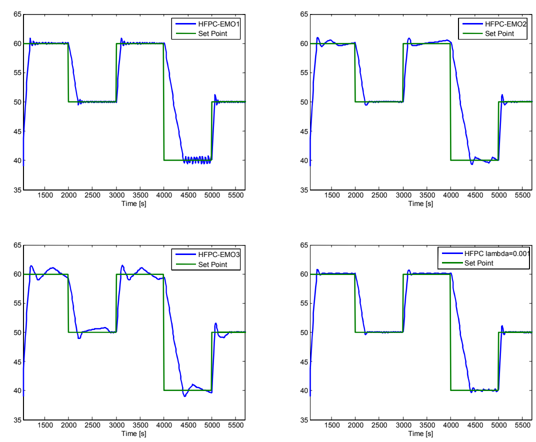

Fig.3. Controlled variable response, criterion 1, 2, 3 and HFPC.

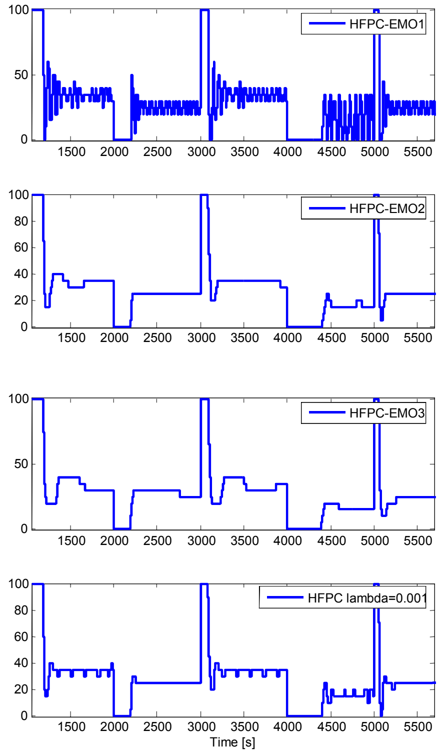

Fig. 4. Simulation test. Manipulated variable responses.

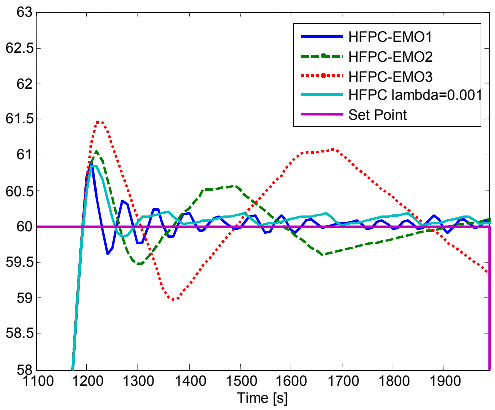

Fig. 5. Controlled variable responses.

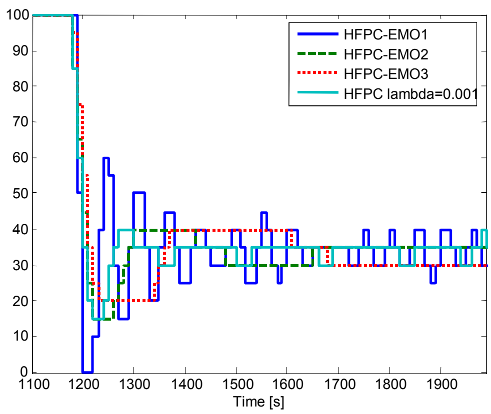

Fig 6. Manipulated variable responses.

Tab. 1 Performance with a Fixed Reference. Mean Values of tracking error and control effort.

	Mean(y-r)^2	Std (y-r)^2	Mean Δu^2	Std Δu^2
EMO-HFPC 1	4.2864	17.5866	118.7500	389.1165
EMO-HFPC 2	4.3693	17.5682	19.6023	76.7000
EMO-HFPC 3	4.6954	17.4941	17.0455	73.4559
HFPC λ=0.001	4.2884	17.5685	25.0000	98.6984

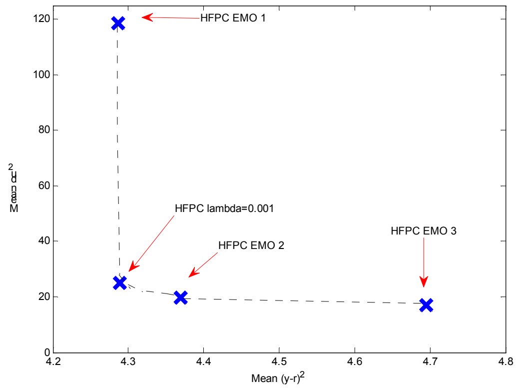

Fig. 7. Pareto front.

3.3 Relation between HPFC-EMO and HPFC

Next the dynamic Pareto front is shown for HFPC-EMO2 as function from instant time between 1000 to 2000 s (Fig 8). For this problem, the Pareto front has different shapes at every instant k as shown in Fig 9.

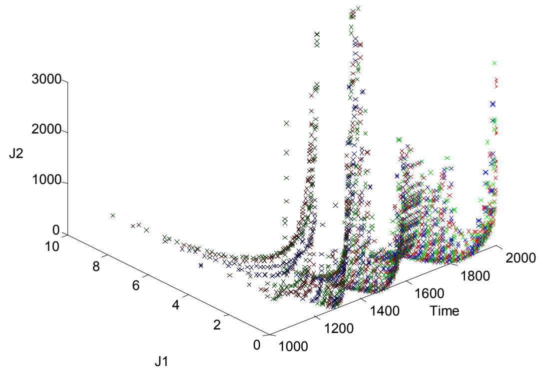

Fig. 8. Dynamic Pareto front, Criterion 2.

From figure 9 and using the analytical LS method described in section 2.4 A), we propose at fixed instant *k*, that Pareto front belongs to the family of curves $J_2 = a_k \cdot J_1^{-b_k}$, with a_k and b_k positive constants parameters at instant *k*. The slope of those curves, evaluated at the optimal objective function values, provides *λ(k)* estimation given by:

$$J_2' = -a_k b_k \cdot J_1^{-b_k - 1} = -\frac{1}{\lambda(k)} \quad (6)$$

Parameters a_k and b_k are obtained by least mean squares at every instant *k*.

Also, few Pareto dominant solutions at some instants are observed (see figure 9, instant 4, 5 and 200). That happens when the optimization problem has activated constraints or the control algorithm has converged. In those cases (P_F have 1 or 2 elements), the IM method 2.4 B) will be considered in order to obtain *λ(k)* values.

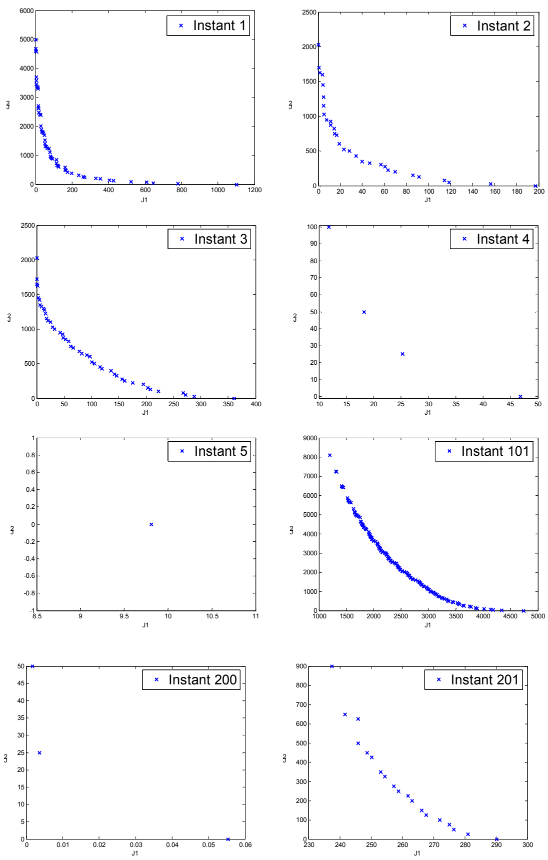

Fig. 9. Dynamic Pareto front, HFPC-EMO2. Each figure represents the Pareto front at one instant.

Figure 10 shows the function *λ(k)* for HFPC-EMO2, determined based on a LS method 2.4 A) and based on the IM method 2.4 B). Note that both estimations are similar.

Fig. 11 shows the evolution for tracking error $|e(k)|$ and control effort $|\Delta u(k-1)|$.

From figures 10 and 11, we realize that there is a relationship between $\lambda(k)$ and $|e(k)|$, $|\Delta u(k-1)|$ at every instant. Thus, two options are proposed to tune the *λ(k)*:

1) By least mean squares based on historical data, to identify the parameters of the following proposed linear model:

$$\lambda(k) = \theta_1 \lambda(k-1) + \theta_2 |e(k)| + \theta_3 \Delta u(k-1).$$

2) *λ(k)* is chosen fixed and equals to the mean value of the signal *λ(k)*.

Table 2 shows the mean value of *λ(k)* and the parameters θ_1, θ_2 and θ_3 of the linear model (option *1)*), obtained for each criterion based on analytical *λ(k)* obtained using LS method. Table 3 also shows the parameters when *λ(k)* is obtained using IM method and option *2)*.

Fig 12 shows *λ(k)* and $\hat{\lambda}(k)$ obtained based on LS (2.4 A) and IM method (2.4 B) for HFPC-EMO2.

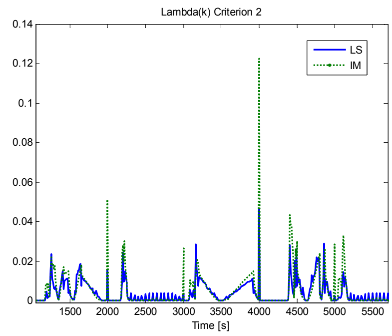

Fig.10. Lambda, HFPC-EMO2. LS and IM.

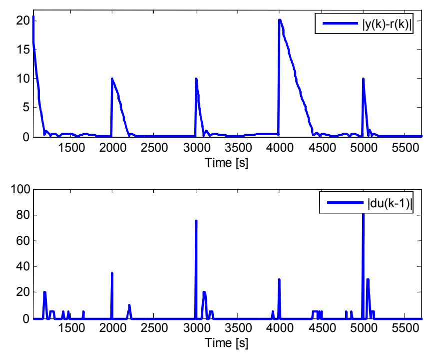

Fig.11. HFPC-EMO2. Tracking error $|y(k) - r(k)|$, control effort $|\Delta u(k-1)|$ indexes.

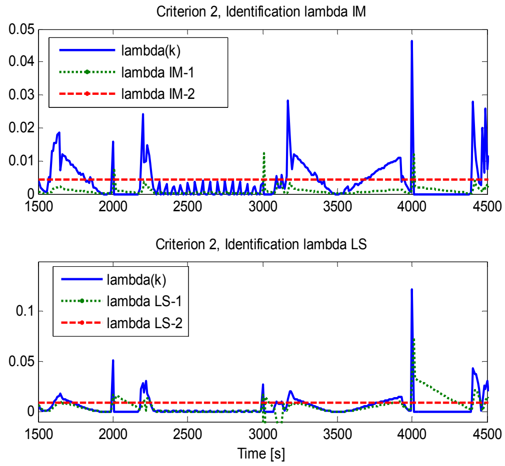

Fig.12. Evolution of $\lambda(k)$ for HFPC-EMO2. LS-1: LS method with option 1), LS-2: LS method with option 2), and IM-1: IM method with option 1), IM-2: IM method with option 2).

Tab. 2 LS method. Mean values of $\lambda(k)$ and parameters for the linear model

	Mean(λ(k))	θ_1	θ_2	θ_3
HFPC-EMO 1	4.2864	17.5866	118.7500	389.1165
HFPC-EMO 2	4.3693	17.5682	19.6023	76.7000
HFPC-EMO 3	4.6954	17.4941	17.0455	73.4559

Tab. 3 IM method. Mean values of $\lambda(k)$ and parameters for the linear model

	Mean(λ(k))	θ_1	θ_2	θ_3
HFPC-EMO 1	0.0074	0.27276	0.0018884	-0.000107
HFPC-EMO 2	0.0086	0.62658	0.0016658	-0.001209
HFPC-EMO 3	0.0182	0.62506	0.62506	-0.001268

Fig 15, Fig 16, Fig 17 and Fig 18 show the system responses using the conventional HFPC algorithm with the tuned lambda obtained from HFPC-EMO2.

Table 4 shows mean values of tracking error and control effort of HFPC using $\lambda(k)$ obtained from HFPC-EMO2 with a fixed reference.

From table 4, the LS method (2.4 A) gives better results than the IM method (2.4 B) due the solutions are very close to the HFPC-EMO2.

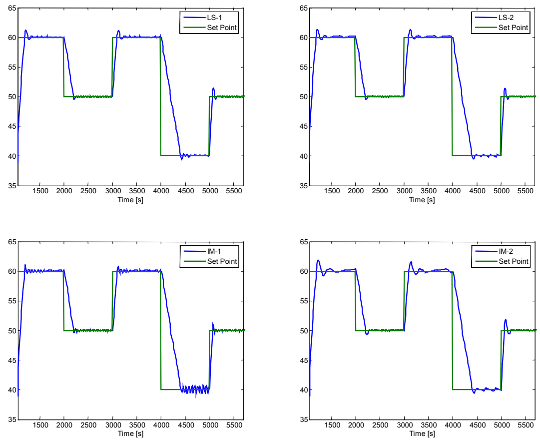

Fig. 15. Controlled variable, LS-1, LS-2, IM-1 and IM-2.

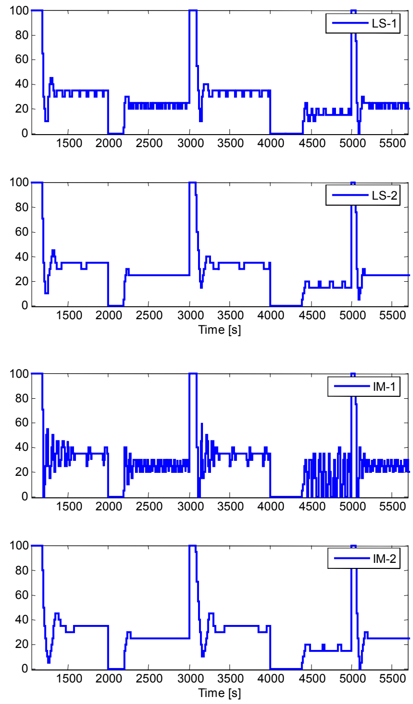

Fig. 16. Manipulated variable, LS-1, LS-2, IM-1 and IM-2.

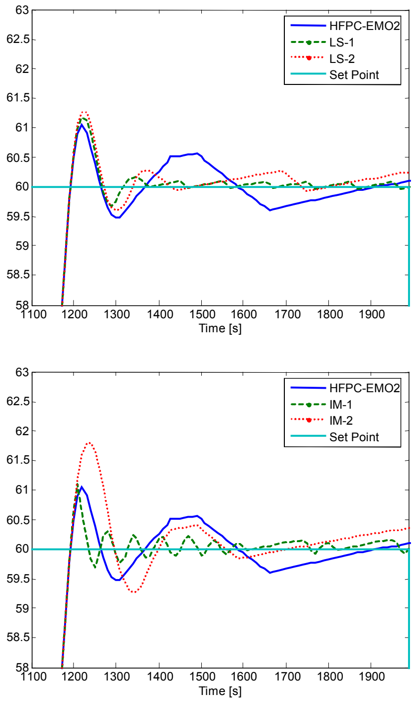

Fig. 17. Controlled variable, LS-1, LS-2, IM-1 and IM-2.

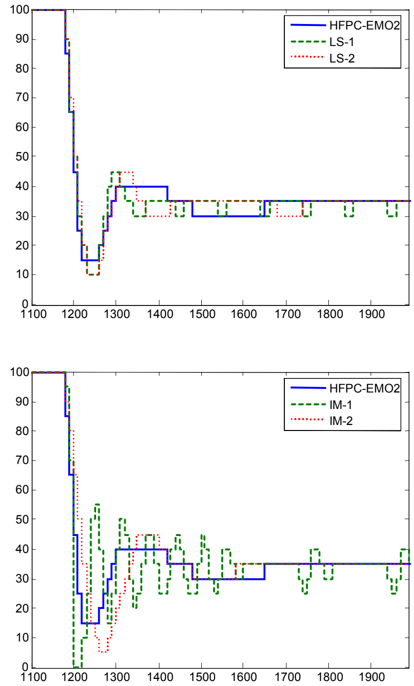

Fig. 18. Manipulated variable, LS-1, LS-2, IM-1 and IM-2.

Tab. 4 Mean values of tracking error and control effort, HFPC-EMO2.

	Mean(y-r)^2	Std (y-r)^2	Mean Δu^2	Std Δu^2
HFPC-EMO 2	4.3693	17.5682	19.6023	76.7000
LS-1	4.3213	17.5792	25.8523	83.9445
LS-1 λ=0.0042	4.3504	17.5727	20.7386	69.5037
IM-1	4.2925	17.5856	108.5227	527.3264
IM-2 λ=0.0086	4.5085	17.5448	16.1932	45.1752

4 Conclusions

This paper presents a new approach of the Hybrid Predictive Control problem by using the Evolutionary Multi-objective Optimization.

We propose two different criteria in order to obtain an optimal control action from the Pareto front. Both criteria are directly related to the tracking error and control effort measurements. This fact could be an efficient tool for the controller designers in real time plants instead of the typical Model Predictive Control.

Thus a tuning method for weighting factor of typical MPC based on the EMO solution was proposed. In this case, two alternatives are considered to obtain the weighting values and we conclude that the model of the Pareto front identified through last mean squares gives the best results.

Further work will be focused on the generalization of the multiobjective predictive control design.

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