IDENTIFICATION OF LONGITUDINAL AND LATERAL DYNAMICS OF AN ULTRALIGHT AIRCRAFT

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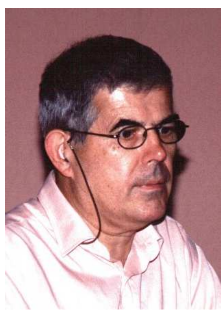
Abstract

Most aircraft can be considered as rigid bodies whose motion is determined by a set of forces due to aerodynamic effects, propulsion and gravity. Their dynamics can thus be described by computing the position and velocity of the center of gravity as well as the orientation and angular velocity of body-fixed axes with respect to a set of earth reference axes. The dynamic equations of motion are well–known so that aircraft modeling leads to very accurate results provided that an equally accurate knowledge of the aircraft parameters and of the acting forces is available. This information is, however, seldom available particularly for light or ultralight aircraft. The alternative to modeling is identification; the practical application of identification techniques is however conditioned by the choice of the class of models (often linear), the design of suitable data–gathering experiments and by the final validation of the obtained model i.e. by the evaluation of the degree of approximation of its description of the real process. This paper describes the identification of the longitudinal and lateral dynamics of an ultralight aircraft and shows that the consistency of the obtained descriptions can heavily depend on the considered class of models. In particular, it is shown that traditional equation-error approaches relying on ARX or those relying on Output Error models can prove unreliable while the less known ARX+noise models can give very consistent results.

Keywords: Identification, Aircraft dynamics, ARX+noise models.

Presenting Author's Biography

Roberto Guidorzi. Roberto Guidorzi holds the chair of System Theory at the University of Bologna since 1980. He has been visiting professor in European and American universities and has collaborated with several industries in the development of advanced projects. He is the author of some 200 publications dealing with subjects of a methodological nature as well as applications. His present research interests concern errors–in–variables identification and filtering, blind channel equalization, aircraft modeling and control and development of e–learning environments.



1 Introduction

The control of aircraft and spacecraft dynamics constitutes an area where most advanced methodologies and technologies find significant applications. The last decade, in particular, has seen the introduction of very sophisticated information and control systems also in light aircraft where it is not uncommon to find systems whose capabilities were previously present only in large commercial or military aircraft. This trend is associated with the use of reliable mathematical descriptions of aircraft dynamics since control loops rely only partially on feedback schemes; the problem of finding accurate models of aircraft is thus, at present, of remarkable importance.

In fact most aircraft can be considered as rigid bodies whose motion is determined by a set of forces due to aerodynamic effects, propulsion and gravity so that very accurate models can be obtained when all aircraft parameters are available. This is certainly true for all modern commercial and military aircraft but the situation is completely different for other categories and it is often impossible to find all relevant data also for very common small general aviation aircraft produced from decades in many thousand pieces.

The alternative approach to classical modeling techniques is identification that consists in selecting, within an assumed class of models, the model that fits more accurately, according to a selected criterion, a set of observations. These procedures can be comparatively simple when applied to processes where no degrees of freedom are available in managing the process input (e.g. macroeconomic systems) but are remarkably more complex when the input to be applied for identification purposes can be designed since it can influence the results. In the case of aircraft dynamics identification, the situation is more complex because the design of the input to be applied must take into account the following aspects [1]: 1) Frequency content suitable for exciting the modes to be described by the model; 2) Limited excursion in order to assure the significance of the obtained model in the neighborhood of the considered point of the flight envelope (speed, thrust, altitude, aircraft configuration); 3) Coding of the input into a sequence of maneuvers to be applied, in a necessarily approximate way, by the pilot; 4) Overall cost of the data gathering flights.

Another critical point of the whole procedure concerns the validation of the obtained models. Limiting our attention to linear models, the possible classes of models that can be selected differ mainly in their description (usually by means of stochastic processes) of the mismatch between models and data and, also when a family of models has been selected, it is still possible to select different models by minimizing different cost functions whose selection should be based on the planned use of the models. When the purpose of identification is, say, the construction of a predictive model, the cost function to be be minimized will be a quadratic function of the prediction error and it will be easy to ascertain the reliability of the model by comparing its prediction with observations. Similar considerations can be repeated for models to be used for simulation purposes. When however the searched models are interpretative ones, the only reliable comparison would be with an accurate description of the system to be identified, i.e. with an unknown information. The strategy that can be adopted in this case could concern: 1) The evaluation of the physical compatibility of the identified models; 2) The comparison of the obtained time constants with those of similar known aircraft with similar size and flight envelope; 3) The evaluation of the congruence of the models identified from data sets obtained in different flights performed in similar conditions. The information gathered in the validation phase can be useful not only to evaluate the suitability of the data gathering phase but also to ascertain the proper choice of the class of models selected for identification.

The purpose of this paper is to describe the identification of models describing the longitudinal and lateral dynamics of an ultralight aircraft (Flight Design CT 2K) for limited pitch ($\pm 4°$) and roll ($\pm 20°$) excursions and in standard cruise conditions at low altitude (1000 ft, flaps at $-12°$ (an uncommon feature of CT), MAP at 25 inches Hg [2]). An interesting aspect of the obtained results concerns the lack of congruence of ARX and Output Error models obtained from different data sets, congruence that has, on the contrary, been observed using ARX+noise models i.e. models where the misfit between model and observations is described by means of both an equation error and an observation noise.

The structure of the paper is the following. Section II recalls the structure of ARX+noise models while Section III describes the results of the identification of the longitudinal dynamics of the aircraft and section IV those concerning lateral dynamics. Short concluding remarks are finally reported in Section V.

2 ARX+noise models

The structure of traditional ARX models is shown in Figure 1 [3, 4] and shows that these models can be partitioned in a deterministic part and in a stochastic one driven by a remote white noise $e(t)$. In fact all equation error models have a similar structure; all that changes is the transfer function of the stochastic part.

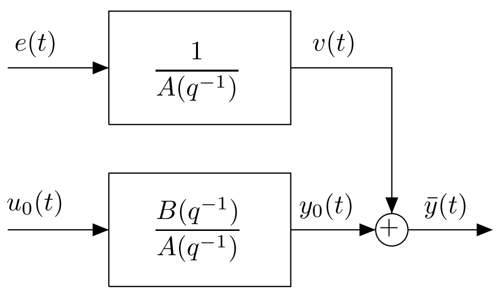


Fig. 1 - Interpretation of ARX models

The input of these models is implicitly assumed as exactly known and the output as affected by the additive colored noise generated by the stochastic part of the model. ARX+noise models are based on a more complex description of the errors that separate the model from the observations; they assume, besides the presence of an equation error, also additive white observation errors on the data as shown in Figure 2 [5, 6].

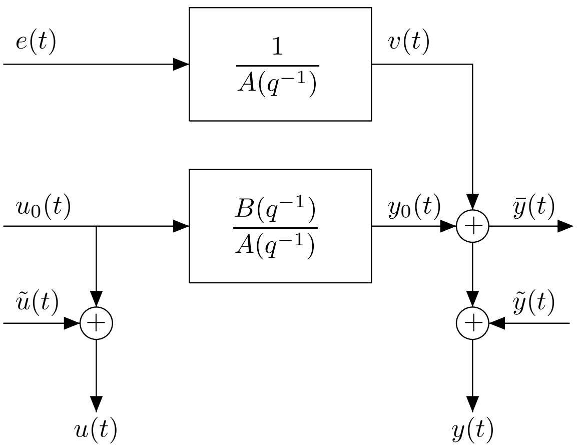


Fig. 2 - Structure of ARX + noise models

These models are uncommon in control and identification but have been used in econometrics where they are designed as “dynamic shock–error models” [7, 8].

ARX+noise models can be described by the difference equation

$$A(q^{-1})\,\bar{y}(t) = B(q^{-1})\,u_0(t) + e(t), \qquad (1)$$

where $u_0(t)$ is the input, $\bar{y}(t)$ the output and $e(t)$ the equation error while $A(q^{-1}), B(q^{-1})$ are polynomials in the backward shift operator q^{-1}

$$A(q^{-1}) = 1 + a_1\,q^{-1} + \cdots + a_n\,q^{-n} \qquad (2)$$

$$B(q^{-1}) = b_1\,q^{-1} + \cdots + b_n\,q^{-n} \qquad (3)$$

and by the relations

$$u(t) = u_0(t) + \tilde{u}(t) \qquad (4)$$

$$y(t) = \bar{y}(t) + \tilde{y}(t) \qquad (5)$$

that describe the link between the noiseless input $u_0(t)$ and the non accessible output $\bar{y}(t)$ and their observations $u(t)$ and $y(t)$. In ARX+noise identification contexts the identification problem can be defined as follows.

Problem 1. Given a set of noisy input–output observations $u(\cdot)$, $y(\cdot)$, determine a consistent estimate of the coefficients a_k $(k = 1, \ldots, n)$, b_k $(k = 1, \ldots, n)$, and of the variances σ_e^2, $\tilde{\sigma}_u^2$, $\tilde{\sigma}_y^2$ of $e(t)$, $\tilde{u}(t)$ and $\tilde{y}(t)$.

The solution of this problem can be reconducted to the solution of an Errors–in–Variables identification problem and, in particular, to the Frisch scheme context [5, 6]. Another possibility could be to rely on Instrumental Variable approaches. These approaches are very simple but are afflicted by a large covariance of the estimates and require, consequently, very long sequences to provide acceptable results; thus they do not look as attractive for the application considered in this paper.

3 Longitudinal model

3.1 Physical model

The aircraft will be considered as a rigid body and the standard NASA coordinate system for body axes will be adopted; in this system x denotes the forward oriented longitudinal axis, y the lateral (right oriented) axis and z the (down oriented) vertical axis. The body axis components of the velocity of the center of mass with respect to earth–based reference axes will be denoted with u, v, w while p, q, r will denote the body axis components of the angular velocity of the aircraft with respect to the reference axes.

The perturbations in the longitudinal and vertical speeds, in the angular velocity along the y axis and in the pitch angle θ introduced by variations in the elevator deflection, δ_e, and in the throttle, δ_t with respect to a steady rectilinear flight can be described by a fourth order state space model of the type

$$\dot{x}(t) = A\,x(t) + B\,u(t) \qquad (6)$$

where $x(t) = [\delta u\ \delta w\ q\ \delta\theta]^T$, $u(t) = [\delta_e\ \delta_t]^T$ and

$$A = \begin{bmatrix} X_u & X_w & -u_0\sin(\theta_0) & -g\cos(\theta_0) \\ Z_u & Z_w & u_0\cos(\theta_0) & -g\sin(\theta_0) \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} X_{\delta_e} & T_{\delta_t}cos(\varepsilon) \\ Z_{\delta_e} & T_{\delta_t}sin(\varepsilon) \\ M_{\delta_e} & 0 \\ 0 & 0 \end{bmatrix}. \qquad (7)$$

In (7) X, Y, Z denote the aerodynamic forces along the body axes and X_u, X_w, X_{δ_e}, Z_u, Z_w, Z_{δ_e} the associated derivatives with respect to u, w and the elevator deflection δ_e at the considered equilibrium point, u_0 and θ_0 are the equilibrium values of the elevator deflection and of the pitch angle, M is the pitch moment and M_u, M_w, M_q, M_{δ_e} the associate derivatives. Finally T denotes the applied thrust and T_{δ_t} its derivative with respect to its variation δ_t and g denotes the acceleration of gravity.

In practice some parameters of this linearized model can be estimated only by performing wind tunnel or specific flight tests. As a reference for a light aircraft model, [9] reports the data of the Navion L–17 (1247 Kg in nominal flight conditions). The short–period mode associated with the higher frequency is $s = -2.51 \pm 2.59\text{j rad/s}$ while the phugoid–mode is $s = -0.017 \pm 0.213\text{j rad/s}$.

3.2 Identified models

A first data set concerns standard cruise conditions at low altitude (1000 ft, flaps at $-12°$, map at 25 inches Hg [2]) on an interval of 400 s. The input is the elevator deflection and the output the pitch angle; its variations, not greater than $\pm 4°$, are reported in Figure 3. A second data set recorded in similar flight conditions and with the same length has been selected for validation purposes; the pitch variations are reported in Figure 4.

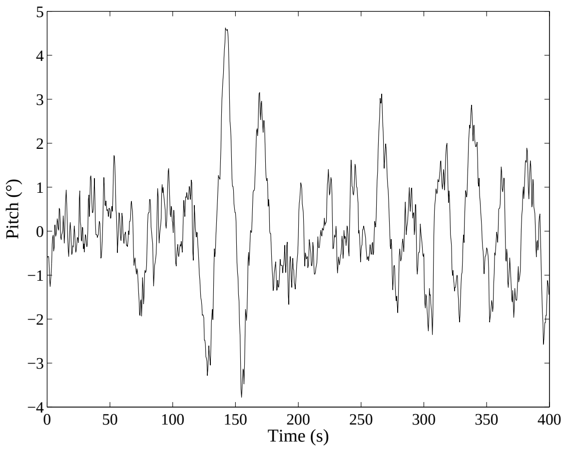


Fig. 3 - Data set 1 output (pitch)

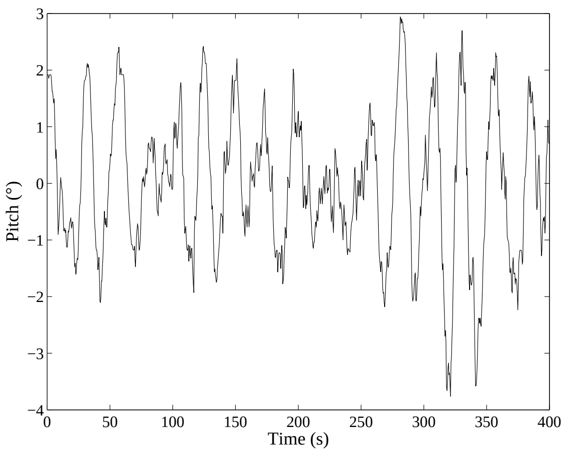


Fig. 4 - Data set 2 output (pitch)

Tab. 1 - Poles of the identified longitudinal models

	Data set 1	Data set 2
ARX + noise	$-0.1725+3.1748i$	$-0.1926+3.2754i$
	$-0.1725-3.1748i$	$-0.1926-3.2754i$
	$-0.1827+0.3617i$	$-0.1776+0.3235i$
	$-0.1827-0.3617i$	$-0.1776-0.3235i$
ARX	$-1.6860+3.1370i$	$-1.8716+3.7519i$
	$-1.6860-3.1370i$	$-1.8716-3.7519i$
	-0.1926	-0.1978
	-2.8948	-1.6939
OE	$-0.7293+3.4555i$	$-0.1537+0.6350i$
	$-0.7293-3.4555i$	$-0.1537-0.6350i$
	$-0.0661+0.3356i$	$-0.0807+0.2405i$
	$-0.0661-0.3356i$	$-0.0807+0.2405i$

Three different classes of models have been tested for identifying the longitudinal dynamics: ARX, Output Error (OE) and the previously mentioned ARX+noise models. The poles of the models identified from the considered data sets (reconducted to the continuous case) are reported in Table 1 where it can be observed that ARX models give reasonably congruent results in the two cases; these models, however, do not describe properly the dynamical behavior of the aircraft since they exhibit a pair of real poles. The output error models are not affected by this problem but the models obtained from the data sets are substantially different. The ARX+noise models are compatible with the physical nature of the system (two pairs of complex poles) and are also comparatively similar. The comparison with the L–17 model shows differences in the short–period and in the phugoid–mode that can be explained with the different masses and inertia moments of the two aircraft. It can be concluded that the stochastic environment described by ARX+noise models is suitable for describing the considered process while this cannot be repeated for ARX and OE models.

4 Lateral model

4.1 Physical model

The linearized dynamic model describing the lateral behavior of an aircraft has the same structure and order of (6) and is described by the matrices [9]

$$A = \begin{bmatrix} Y_v & -u_0\cos(\theta_0) & u_0\sin(\theta_0) & g\cos(\theta_0) \\ N_v & N_r & N_p & 0 \\ L_v & L_r & L_p & 0 \\ 0 & \tan(\theta_0) & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ L_{\delta_a} & L_{\delta_r} \\ 0 & 0 \end{bmatrix} . \qquad (8)$$

The state is now $x(t) = [v\ r\ p\ \phi]^T$, where ϕ is the roll angle and the input is $u(t) = [\delta_a\ \delta_r]^T$ where δ_a and δ_r denote the aileron and rudder deflections with respect to the considered equilibrium values. L and N are the roll and yaw moments and L_v, L_r, L_p, L_{δ_a}, L_{δ_r}, N_v, N_r, N_p, N_{δ_a}, N_{δ_r} the associated derivatives. Y_v, Y_{δ_a} and Y_{δ_r} denote the derivatives of Y with respect to v, δ_a and δ_r.

The values of the modes reported in [9] for the L–17 are $s = -8.433$ rad/s for the roll mode, $s = -0.0088$ rad/s for the spiral mode and $s = -0.486 \pm 2.334$j rad/s for the dutch roll mode.

4.2 Identified models

A first data set collected in the same standard cruise conditions considered for the longitudinal model and observed over a period of approximately 200 s has been selected. The input is the ailerons deflection and the output the roll angle (Figure 5).

A second data set recorded in similar conditions and with the same length has been selected for validation purposes; the roll angle variations are reported in Figure 6. The excursion of the roll angle in these data sets does not exceed $\pm 20°$.

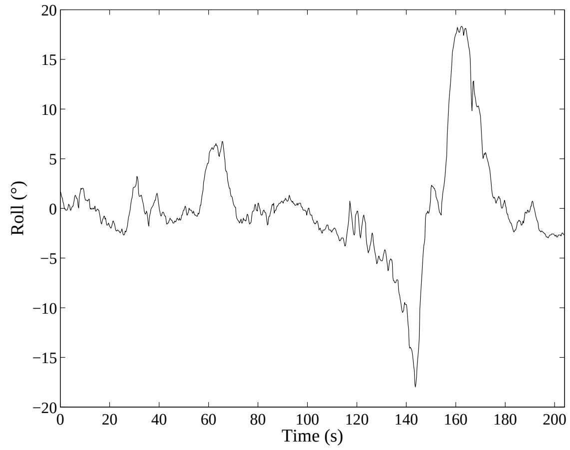


Fig. 5 - Data set 1 output (roll)

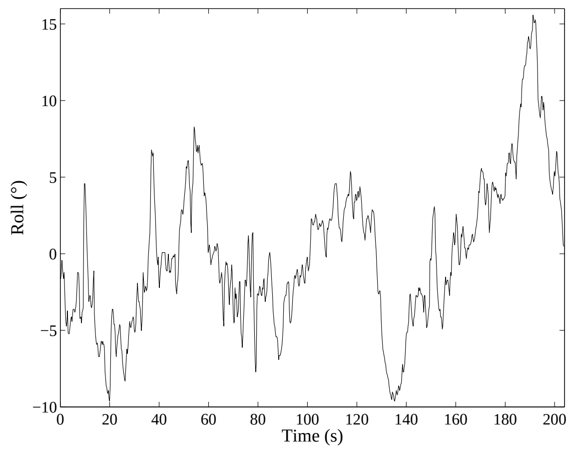


Fig. 6 - Data set 2 output (roll)

Tab. 2 - Poles of the identified lateral models

	Data set 1	Data set 2
ARX + noise	$-2.2699 + 6.6662i$	$-2.0583 + 6.1859i$
	$-2.2699 - 6.6662i$	$-2.0583 - 6.1859i$
	-0.0591	-0.0647
	-1.3519	-1.7720
ARX	$-4.3121 + 8.5431i$	$-5.7829 + 6.5862i$
	$-4.3121 - 8.5431i$	$-5.7829 - 6.5862i$
	-0.0511	-0.0501
	-2.0469	-3.9869
OE	$-0.6457 + 3.4370i$	$0.0152 + 0.0621i$
	$-0.6457 - 3.4370i$	$0.0152 - 0.0621i$
	$-0.0972 + 0.1596i$	-0.0734
	$-0.0972 - 0.1596i$	unstable pole

Like in the case of longitudinal models, the classes considered for identification are ARX, Output Error (OE) and ARX+noise models. The poles of the models identified from the considered data sets (reconducted to the continuous case) are reported in Table 2.

It is possible to observe that, in this case, OE models are not congruent over the two data sets; these models are also not congruent with respect to the physical model.

Better results are given by ARX models that exhibit an acceptable degree of congruence over the considered sets and also with respect to the physical structure of the process.

The best results are however given, as in the case of longitudinal models, by ARX+noise models that exhibit an high degree of congruence over the different data sets and give values compatible with the characteristics of the tested aircraft.

It can be concluded that the stochastic environment described by ARX+noise models leads to the best identification also of the lateral dynamics.

Fig. 7 - The CT 2K during data collection

5 Concluding remarks

This paper has described the identification of dynamical models of the longitudinal and lateral behavior of an ultralight aircraft by means of different classes of models.

The cross–validation of the models over two different data sets has shown that the best results have been obtained by using ARX+noise models i.e. ARX models where unknown amounts of additive white noise is assumed on the input and output observations. Other classes of models traditionally used in aircraft identification like ARX and Output Error ones [1] have given poor and/or non consistent results.

Since the obtained models derive from linearizations, their validity is restricted to flight conditions similar to those concerning the data collection that has considered limited pitch ($\pm 4°$) and roll ($\pm 20°$) variations and standard cruise conditions at low altitude (1000 ft, flaps at $-12°$, MAP at 25 inches Hg).

Despite these limitations, these models can be useful both for interpretative purposes, i.e. to obtain a guess of some physical parameters that could be measured only by means of complex and expensive setups, and also for designing automatic control devices like autopilots.

Other applications can concern the transversal and longitudinal dynamic stability analysis of an aircraft as performed in [10].

6 References

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