HYSTERESIS MODELLING FOR A MR DAMPER

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Abstract

An experimental dataset of a commercial Magneto-Rheological (MR) damper is exploited for identification of a Hysteresis-based Control-Oriented model. The model wellness for hysteresis, saturation and transient responses is shown through validation with experimental data. A study case that includes a Quarter of Vehicle (QoV) shows that the hysteresis phenomena could affect the primary ride and vehicle handling. Several analysis based on open and closed loop simulation demonstrated that hysteresis must be considered for controller design.

Keywords: hysteresis, MR damper model, model simulation, vehicle dynamics

Presenting Author’s Biography

Ruben Morales-Menendez holds a PhD Degree in Artificial Intelligence from Tecnologico de Monterrey. From 2000 to 2003, he was a visiting scholar with the Laboratory of Computational Intelligence at the University of British Columbia, Canada. For more than 23 years, he has been a consultant specializing in the analysis and design of automatic control systems for continuous processes. He is a member of the National Researchers System of Mexico (Level I) and a member of IFAC TC 9.3.
1 Introduction

The Magneto-Rheological (MR) damper is a non-linear component with dissipative capability used in the control of semi-active suspensions, where the damping coefficient varies according to the applied electric current. The modelling of the force-velocity curve is not a trivial task due to the hysteresis loop and nonlinear behavior. The MR damper inherits the hysteresis and bilinear behavior from its MonoTube (MT) mechanical design. Several MT damper modelling approaches show that the acceleration $\ddot{z}_{def}$ and velocity $\dot{z}_{def}$ as input variables are needed for a precise hysteresis simulation, [1, 2], and for a realistic simulation of chassis acceleration, suspension deflection and road holding, [3, 4]. A MR damper model including the hysteresis becomes necessary in order to obtain precise simulations.

This paper presents a MR damper model based on the arrangement of two devices that models the passive force consisting of a spring and a damper, and a damper which models the force due to the current effect over the oil viscosity. The model captures the hysteresis and the current effect achieving frequency independent parameters. The study of the hysteresis compliance is done through a comparison of two modelling approaches in the suspension of a Quarter of Vehicle (QoV).

This document is organized as follows. A theoretical background on MR damper modelling and its challenging issues are exposed in section 2. Section 3 presents the proposed model, and experimentation and identification procedure. A study case is presented in Section 4. Section 5 discusses the findings and results. The conclusion is presented in section 6. Table 1 defines the paper nomenclature.

2 Theoretical background

Some fundamental concepts are reviewed, [5]:

(a) When a certain magnetic field is applied to the MR fluids, the particles in the fluids are polarized and they form polarization chains, in the parallel direction to the applied field.

(b) When the MR fluids are subjected to an external shear stress in the perpendicular direction to the magnetic field, those polarization chains can resist the shear stress to some extent, and the MR fluids behave in a viscoelastic way. This region is referred to as the pre-yield region. When the external shear stress is increased and exceeds a certain value, the polarization chains will be broken and MR fluids becomes in regular Newtonian fluids; this region is referred to as the post-yield region.

(c) The gradually decreasing shear stress links up the polarization chains, but the stress value to do the link is less than before the polarization chains break happens, producing the hysteresis. The switch of MR fluids from viscoelastic behavior to regular Newtonian fluid behavior is referred to as yield, and the value of the external shear stress at this point is thus known as yield stress, and depends on the intensity of the applied field. Since the polarization chains are increased with the magnetic

<table>
<thead>
<tr>
<th>Tab. 1 Nomenclature</th>
<th>$a_i$</th>
<th>Coefficient depending polynomially on the current</th>
<th>$c_{MR}$</th>
<th>MR damping coefficient in $(N \cdot A)/m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_p$</td>
<td>Passive damping coefficient in $N s/m$</td>
<td>$h_h$</td>
<td>Coefficients dealt with the horizontal hysteresis</td>
</tr>
<tr>
<td></td>
<td>$k_p$</td>
<td>Passive stiffness coefficient in $N/m$</td>
<td>$f_{MR}$</td>
<td>MR damping observed force in $N$</td>
</tr>
<tr>
<td></td>
<td>$f_p$</td>
<td>Force due to damper mechanics ($N$)</td>
<td>$f_M$</td>
<td>Predicted force</td>
</tr>
<tr>
<td></td>
<td>$h$</td>
<td>Input magnetization rate into a ferromagnetic material</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$c_2, c_3$</td>
<td>Magnetic saturation due to remanent magnetism and minor loops</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_+$</td>
<td>Positive magnetic saturation level for positive $h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_-$</td>
<td>Negative magnetic saturation level for negative $h$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_s$</td>
<td>Coefficient for the minor loops and dc bias</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$h_{s1}, h_{s2}, h_{s3}$</td>
<td>Coefficients for the hysteresis</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$h_v$</td>
<td>Coefficient for vertical hysteresis</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$I$</td>
<td>Applied current in A</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$q$</td>
<td>Exponent for $f$ in the polynomial</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$r$</td>
<td>Polynomial order for the polynomial current dependency</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$y_1, y_2$</td>
<td>Coefficients for the MR sigmoid behavior</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x, z, z_{def}$</td>
<td>Piston deflection in meters</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x, z, z_{def}$</td>
<td>Piston deflection velocity in m/s$^2$</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$x_v$</td>
<td>Chassis velocity in m/s</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$w$</td>
<td>Disturbation shaped as chirp sinusoidal</td>
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<tr>
<td></td>
<td>$\alpha_{q,i}$</td>
<td>$i$-th coefficient for the polynomial current equation</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$\theta$</td>
<td>Parameters vector</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$\phi_{dataest}$</td>
<td>the input dataset matrix</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t$</td>
<td>$T_{in}$ input data row</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Z_{def}, Z_{def}$</td>
<td>Column vectors of the experimental dataset</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Z_{def}, f$</td>
<td>experimental dataset</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\omega$</td>
<td>Frequency in rad/s</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$f_{XX-MR}$</td>
<td>MR damping force estimation. The letters XX can be CO or HCO</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f_{steering}$</td>
<td>Force generated for a steering action</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$f: a \rightarrow b$</td>
<td>The function $f$ maps the element $a$ to the element $b$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$R$</td>
<td>Sinusoidal amplitude in m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
field, the shear stress need to be large in order to break them. The larger the magnetic field, the larger the yield force.

A MR damper is usually characterized by the displacement and/or velocity of the piston, the electric current applied to the coil as inputs and the force generated on the piston as output. A static MR damper model is build having direct or instantaneous links between input variables, thus it estimates the force using the present current and displacement/velocity and not earlier values. A dynamic MR damper model is also built with the same variables but these variables change before affecting the force value by their intrinsic dynamics, and their values will thereby also depend on early current or displacements.

A classification for this device is proposed by its original structure as: passive and I-driven. A passive original model does not include the electric current as input variable while the I-driven model does. In the literature, the MR damper modelling has been based in all the known model structures: phenomenological (parameters which meaning is related to the mechanical parts, physical meaning), semi-phenomenological (parameters with physical meaning) and the black-box support.

2.1 Passive models

The MR damper is often described by the general form:

$$f_p = \left[ g(\dot{x}, x) \mid \theta \right]$$

where the fluid is MR. Therefore, the study of the static curves force-velocity and the transient response have been the basis of such models. The consequence is the missing of the electric current (or voltage) as a model input, Fig. 1.

![Fig. 1 Passive MR damper model](image)

The electric current effect on the damping force is addressed by the representation of each model parameter as a polynomial function of the current; hence, the model is represented as

$$f_{MR} = \left[ g(\dot{x}, x) \mid \theta(I) \right] \quad a_i(I) = \sum_{q=0}^{r} \alpha_{q,i} \cdot I^q(I)$$

Consequently, there is an over-parameterization, and if the polynomial order is greater than 1, the nonlinear complexity is augmented. The main properties of the model are not improved. There are several important works: Bingham model [6], Bouc Wen Modified [7], Polynomial Approach [8], Three Parameters model [9], Sigmoidal-based Behavior model [10], and Phase-Transition model [5]. Table 2 compares some works in this field.

2.2 I-driven models

The MR damper is described by

$$f_{MR} = \left[ g(\dot{x}, x, I) \mid \theta \right]$$

This model considers the applied current as a model input variable, Fig. 2.

![Fig. 2 I-driven MR damper model](image)

The parameter identification requires of experiments obtained with persistent excitation in both displacement and current. The modelling contributions are classified as Surface Response Method or Statistical model [11], the Nonlinear Auto-Regressive with eXogenous input (NARX) model [12], NARX based in Neural Networks (NNARX) [13], considered as black box or non-parametric models, and the Dynamic Transfer function Current-Force [14], which analyzes the transient behavior of the force due to the electric current. Table 3 compares some features of those approaches.
hysteresis loop, the translation of the
In order to develop the mathematical expression for a
with sigmoid T-shape
Equation (3) can represent a magnetic saturation curve
or remanent magnetism. The minor loops, is a collective name of all the
loops, which have at least one extreme different from the
major loop extreme. When the exciting magnetic
field is interrupted during the process of magnetization,
then the magnetization of the sample will not stay the
same, it declines to a value below the value determined
by the field at the point of the interruption on the hysteresis loop, phenomenon called remanent magnetism.

The Force-Velocity (FV) damper map is a well known
diagram with high complexity. It exposes the damping
ratio \( \zeta \) behavior. If \( \zeta \) has a one-to-one relation, the next sentence is true, based on (3).

Let \( \zeta : \dot{z} \mapsto h, \quad f_{MR} \mapsto T(h) \) be defined by
\[ \zeta(\dot{z}, f_{MR}) = f_{MR}/\dot{z}. \]
represents the damper behavior without hysteresis and constant current. In order to obtain a more precise mathematical approach of a damper, the following assumptions and analogies are proposed.

By assuming a harmonic motion of the damper piston,
let \( z, \dot{z}, \ddot{z} \) approximated by:

\[ z \sim R \cdot \sin(\omega \cdot t) \quad (6) \]
\[ \dot{z} \sim \omega \cdot R \cdot \cos(\omega \cdot t) \quad (7) \]
\[ \ddot{z} \sim -(\omega^2) \cdot R \cdot \sin(\omega \cdot t) \quad (8) \]

It has been shown that the acceleration deflection \( \ddot{z} \) is the stronger influence on shaping of the hysteresis, [1, 2]. The \( \ddot{z}_{def} \) inertial effect contributes to the hysteresis offset. From equations (4, 5), and based on [1, 2, 10, 16], the static mapping defined in [15] is extended to a dynamic mapping

\[ H_h : h_h \cdot z \mapsto a_{0\text{dynamic}} \quad (9) \]
\[ H_i : h_i \cdot \dot{z} \mapsto c_{I,3\text{dynamic}} \quad (10) \]
\[ H_v : h_v \cdot \ddot{z} \mapsto b_{1\text{dynamic}} \quad (11) \]

Thus, the values for the coefficients defined in (4) and
(5) will change according to \( z, \dot{z} \) and \( \ddot{z} \). The hysteresis loop will be shaped depending on the piston motion dynamics. The proposed model is defined by:

\[ f_{MR} = k_p \dddot{z} + c_p \dot{z} + h_v \cdot \ddot{z} + \]
\[ h_{s2} \cdot \tanh(h_{s3} \cdot \dot{z} + h_h \cdot z) + \]
\[ c_{MR} \cdot I \cdot \tanh(y_1 \cdot \dot{z} + y_2 \cdot \ddot{z}) \quad (12) \]

where the measure of the horizontal shift \( a_0 \) represents
the value of the coercive force, where the hysteresis loop intersects the horizontal axis. The value of \( b_1 \) is determined by \( h_{max} \), the maximum value of \( h \) where the two curves \( f_+ \) and \( f_- \) intersect. The values of \( c_2 \) and \( c_3 \) represent the minor loops and the remanent magnetism.

3 MR damper modelling

A general MR damper structure model can be described by
two dampers in parallel: a damper with constant shear stress (passive) and a damper with variable-shear stress (semi-active) due to the variation of the applied electric current. The sum of these components yields the MR total damping force \( f_{MR} \):

\[ f_{MR} = f_p(z, \dot{z}) + f_I(z, \dot{z}, I) \]
\[ f_p = k_p \dddot{z} + c_p \dot{z} + a(z, \dot{z}, \ddot{z}) \]
\[ f_I = c_{MR} \cdot I \cdot g(z, \dot{z}) \quad (2) \]

where \( a(\cdot) \) and \( g(\cdot) \) are nonlinear functions. \( a(\cdot) \) describes hysteresis and \( g(\cdot) \) describes the semi-active behavior.

3.1 Hysteresis-based Control-Oriented (HCO) Model

Equation (3) can represent a magnetic saturation curve with sigmoid T-shape \( T \) for a \( h \), [15].

\[ T(h) = A_0 \cdot h + B_0 \cdot \tanh[C_0 \cdot h] \quad (3) \]

In order to develop the mathematical expression for a hysteresis loop, the translation of the \( T(h) \) function horizontally is done by \( \pm a_0 \) as vertically by \( \pm b_1 \). The sign \( \pm \) specifies directions,

\[ T(h) = A_0 \cdot h + B_0 \cdot \tanh[C_0 \cdot h - a_0] + b_1 + c_2 \quad (4) \]
\[ T(h) = A_0 \cdot h + B_0 \cdot \tanh[C_0 \cdot h + a_0] - b_1 + c_3 \quad (5) \]

Tables 2 and 3 compare the main features and characteristics of MR damper models. Based on this comparison, some demands are found: (a) the hysteresis phenomena has not been addressed with success given that the analytical models are based on static force-velocity curves where the applied electric current is constant and they lack of the acceleration \( \ddot{z} \) as input, an important inertial element in the damping force [1]. (b) If the experimental database has static currents, the current could be defined as a varying parameter, where the continuous-time variation of this parameter is considered. (c) there is a necessity for a control-oriented model given that the literature models are proper for estimation or diagnosis and their application for controller synthesis becomes involved.

<table>
<thead>
<tr>
<th>Features</th>
<th>NARX</th>
<th>NNARX</th>
<th>Statistical</th>
</tr>
</thead>
<tbody>
<tr>
<td>Application</td>
<td>e</td>
<td>e</td>
<td>d</td>
</tr>
<tr>
<td>( \theta ) length</td>
<td>12</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Hysteresis</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( f ) as input</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Type</td>
<td>Dyn</td>
<td>Dyn</td>
<td>Static</td>
</tr>
<tr>
<td>Frequency dependent parameters</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Inputs</td>
<td>( f(I, z, \dot{z}) )</td>
<td>( f(I, z, \dot{z}) )</td>
<td>( f(I, z, \dot{z}) )</td>
</tr>
<tr>
<td>Fitting method</td>
<td>LSM</td>
<td>Levenberg-Marquardt</td>
<td>Surface Response</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type</th>
<th>Features</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>Application</td>
<td>e</td>
</tr>
<tr>
<td>A</td>
<td>( \theta ) length</td>
<td>12</td>
</tr>
<tr>
<td>A</td>
<td>Hysteresis</td>
<td>Yes</td>
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<tr>
<td>A</td>
<td>( f ) as input</td>
<td>Yes</td>
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<tr>
<td>A</td>
<td>Type</td>
<td>Dyn</td>
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<tr>
<td>A</td>
<td>Frequency dependent parameters</td>
<td>No</td>
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<tr>
<td>A</td>
<td>Inputs</td>
<td>( f(I, z, \dot{z}) )</td>
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<tr>
<td>A</td>
<td>Fitting method</td>
<td>LSM</td>
</tr>
</tbody>
</table>

Same nomenclature as in Table 2.

**Tab. 3 Classification of the literature I-driven models.**

<table>
<thead>
<tr>
<th>Type</th>
<th>Features</th>
<th>Description</th>
</tr>
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<tbody>
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<td>Application</td>
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</tr>
<tr>
<td>A</td>
<td>( \theta ) length</td>
<td>12</td>
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<tr>
<td>A</td>
<td>Hysteresis</td>
<td>Yes</td>
</tr>
<tr>
<td>A</td>
<td>( f ) as input</td>
<td>Yes</td>
</tr>
<tr>
<td>A</td>
<td>Type</td>
<td>Dyn</td>
</tr>
<tr>
<td>A</td>
<td>Frequency dependent parameters</td>
<td>No</td>
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<tr>
<td>A</td>
<td>Inputs</td>
<td>( f(I, z, \dot{z}) )</td>
</tr>
<tr>
<td>A</td>
<td>Fitting method</td>
<td>LSM</td>
</tr>
</tbody>
</table>

**Note:**

- The static mapping defined in [15] is extended to a dynamic mapping.
- The acceleration deflection \( \ddot{z} \) is the stronger influence on shaping of the hysteresis, [1, 2]. The \( \ddot{z}_{def} \) inertial effect contributes to the hysteresis offset. From equations (4, 5), and based on [1, 2, 10, 16], the static mapping defined in [15] is extended to a dynamic mapping.
3.2 Control-Oriented (CO) model structure

Modifying the Semi-Phenomenological model [10], an I-driven model (13) can be proposed. This modelling approach performs good simulations of the bi-viscous and saturation behavior but limits the hysteresis representation.

\[ f_{MR} = c_{MR} \cdot I \cdot \tanh (y_1 \cdot \dot{z} + y_2 \cdot z) + c_p \cdot \dot{z} + k_p \cdot z \]  (13)

3.3 Experimental data and identification

A Frequency modulated displacement with ICPS electric current as input signal was exploited in order to identify the MR damper in a experimental system, [16]. The models represented by equations (12,13) are identified by using the nonlinear least square curve fit algorithm with the following cost function:

\[ \min_\theta ||f_{MR}(\theta, \phi_{dataset}) - f_{MR}||^2 = \min_\theta \sum_i \left( f_{MR}(\theta, \phi_i) - f_{MR_i} \right) \]

where \( \phi_{dataset} = [z_{def}, \dot{z}_{def}, \ddot{z}_{def}, I] \).

4 Study case

The system consists of a simple model of a Quarter of Vehicle (QoV). The main components are the sprung mass \( (m_s) \) and the unsprung mass \( (m_{us}) \). The spring with a stiffness coefficient \( k_s \) and a semi-active damper built the suspension between masses. The stiffness coefficient \( k_t \) models the wheel tire. The vertical position of the mass \( m_s \) \( (m_{us}) \) is defined by \( z_s \) \( (z_{us}) \). It is assumed that the wheel contact is kept.

The used QoV parameters corresponds to a Renault Megane Coupe\(^{TM}\) model (see [17]) whose values are: \( m_s = 315 \text{ Kg} \), \( m_{us} = 37.5 \text{ Kg} \), \( k_s = 29500 \text{ N/m} \), \( k_t = 210000 \text{ N/m} \). The damper parameters are defined according to equations. (13) and (12).

Two QoV simulation systems with a semiactive suspension were implemented. They used MR damper models (12) and (13) resulting equations (15) and (17), called linear QoV and nonlinear QoV respectively. The steering force is considered zero. The linear word is used in the sense of the hysteresis phenomena presence on the MR damper.

The dynamical equations for the linear QoV are governed by:

\[ m_s \ddot{z}_s = -k_s \cdot z - f_{CO-MR} + f_{steering} \]
\[ m_{us} \ddot{z}_{us} = k_s \cdot z + f_{CO-MR} - k_t \cdot (z_{us} - z_r) \]  (14)

where the \( f_{CO-MR} \) is estimated by:

\[ f_{CO-MR} \approx c_p \cdot \dot{z} + k_p \cdot z \\
+ c_{MR} \cdot I \cdot \tanh (y_1 \cdot \dot{z} + y_2 \cdot z) \]  (15)

and it uses the parameters for the CO MR damper model shown in Table 4.

The dynamical equations for the nonlinear QoV are governed by:

\[ m_s \ddot{z}_s = -k_s \cdot z - f_{HCO-MR} + f_{steering} \]
\[ m_{us} \ddot{z}_{us} = k_s \cdot z + f_{HCO-MR} - k_t \cdot (z_{us} - z_r) \]  (16)

where the \( f_{HCO-MR} \) is estimated by:

\[ f_{HCO-MR} \approx k_p \cdot \dot{z} + c_p \cdot \dot{z} + h_v \cdot \dot{z} + h_b \cdot z \\
+ c_{MR} \cdot I \cdot \tanh (y_1 \cdot \dot{z} + y_2 \cdot z) \]  (17)

and it uses the parameters for the HCO MR damper model shown in Table 4.

It is worth to note that when the applied current equals 0 A, the passive damping is approached by a linear coefficient in equation (15). The frequency response or pseudo-bode, [18], of both systems exposes the controllability margins with and without hysteresis inclusion.

Two simulation tests were done: open and closed control systems, Figure 3.

![Fig. 3 Diagram of simulation for the quarter of vehicle in open and closed loop. The controller input signals are considered measurable. The filter block is required in order to assure soft changes when the control law switches between the commanded current values, defined in (18). The filter cut-off frequency must be at least 20 Hz in order to cover the automotive bandwidth.](image)

The applied current was \( \{0.001, 0.5, 1, 1.5, 2, 2.5\} \) A for the open control system. For the closed control system, comfort oriented controller Power-Driven-Damper Control (PDD), [19], evaluates both systems. The specifications for the comfort, [20], are defined in the span of [0-10] Hz, the maximum gain of the frequency response \( \dot{z}_s/z_r \) must be kept low below 200. The (PDD) control strategy is obtained from control of Hamiltonian port systems and claims for soft manipulations and good performance under \( \omega = 120 \text{ rad/s} \).
The closed control input is the current, hence, the control strategy has been modified in order to be proper for the MR QoV suspension. The modified PDD control law is:

\[
\begin{align*}
I_{\text{min}} & = \left\{ \begin{array}{ll}
K \cdot z_{\text{def}} \cdot z_{\text{def}} + b_{\text{max}} \cdot z_{\text{def}}^2 < 0 & \\
I_{\text{max}} \quad & \text{otherwise}
\end{array} \right. \\
I_{\text{max}} & = \left\{ \begin{array}{ll}
K \cdot z_{\text{def}} \cdot z_{\text{def}} + b_{\text{min}} \cdot z_{\text{def}}^2 & \geq 0 \\
(I_{\text{max}} + I_{\text{min}})/2 & \text{if } z_{\text{def}} = 0 \quad \text{and} \quad z_{\text{def}} \neq 0 \\
I_{\text{max}} \cdot z_{\text{def}}/z_{\text{def}} & \text{otherwise}
\end{array} \right.
\]

where \( K = k + k_p, b_{\text{max}} \) corresponds to the MR damper model force evaluated in \( z_{\text{def}} \) at maximum current 2.5 A and divided between \( z_{\text{def}} \) m/s, \( b_{\text{min}} \) corresponds to the MR damper model force evaluated in \( z_{\text{def}} \) at zero current and divided between \( z_{\text{def}} \) m/s. The values for \( I_{\text{min}} \) and \( I_{\text{max}} \) where defined as 0.2 and 2.5 A. See Figure 3 for the simulation diagram and filter frequencies for the controller.

5 Discussion

Table 4 shows the identified parameters.

<table>
<thead>
<tr>
<th>Model</th>
<th>( c_{\text{MR}} )</th>
<th>( c_p )</th>
<th>( h_p )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CO</td>
<td>441</td>
<td>636</td>
<td>-20356</td>
<td>12</td>
<td>14.3</td>
</tr>
<tr>
<td>HCO</td>
<td>444</td>
<td>246</td>
<td>5977</td>
<td>7.12</td>
<td>7.9</td>
</tr>
</tbody>
</table>

The descriptive force-velocity curves shows the good tracking of the hysteresis with the model HCO, left top plot in Fig. 4. The model CO makes good saturation estimation but at low velocity the precision is small, right top plot in Fig. 4. The force-velocity curves, bottom plot in Fig. 4, exposes the covering done by each model versus real data where the real hysteresis matches in a more precise shape for model HCO.

In transient response, the model HCO becomes more accurate for small and big forces. The model CO fails the estimation for small forces (< 200 N) because of the inertial effects are missing the model. See the models error at low forces on Fig. 5.

The classification of the evaluated models shows better features than the aforementioned reviewed. Table 5, adding an easier process for their identification. A wide analysis is necessary in order to define the frequency dependence of the parameters.

The open loop tests exposes a different controllability when the suspension includes hysteresis and when it does not, see Fig. 6. In the span of primary ride (0-2.5 Hz) the nonlinear case shows a more realistic and constrained controllability than its linear counterpart. For maximum current and rattle-space frequency (the chassis and the tire fixture can shock each other, approx. 1.8 Hz), referring to the linear case, the \( \tilde{z}_{\text{def}} \) shows very low gain and very good comfort while the nonlinear case is more conservative. In the spans of (2.5-11 Hz) and (16-20 Hz) both approaches offer similar performances, but the minimum gain is greater with hysteresis. In (11-16 Hz), the simulation results show how the tire-hop (the tire can be separated from the surface) frequency is affected by the hysteresis being an interesting and important result in the evaluation on controllers, given that this frequency is key for the vehicle handling. The simulation allows to infer that in this bandwidth, high currents are desired in order to improve the vehicle stability.

The closed loop test offers more illustrative results. The controller is very efficient when the semi-active suspension does not include the hysteresis (similar to that reported in [19]). When the hysteresis is taken into account, i.e. the MR damper is HCO model, the controller performs as expected for 2-8 Hz but for the rattle-space frequency the performance is very poor from the obtained without hysteresis.
Fig. 5 MR damper models transient response

Fig. 6 Performance of semi-active suspensions, with and without hysteresis, in order to modify the control objective (comfort) in a semi-active suspension in the span of 0-2.5 A of current. Top plot is the linear QoV case, bottom plot is the nonlinear QoV case.
6 Conclusion

This work proposes a MR damper model called (HCO) with several features: a simple structure, standard identification procedure, precision in hysteresis simulation, and proper structure for controller synthesis. This modelling approach is compared with a model based on transient response. A study case, a Quarter of Vehicle with a MR damper shows that the hysteresis could affect the primary ride and vehicle handling. Missing the hysteresis phenomenon decreases the negative effects on the rattle-space and tire-hop frequencies. Hence, this work focuses the attention to more accurate specification in the simulation of vehicle behavior for controller evaluation purposes. The example was illustrative and requires more analysis.

7 References


